Problems to the lecture

Fundamentals of Electrical Engineering

For Students of the Bachelor Courses

- **Elektrotechnik Jülich ET**
- **Electrical Engineering ET-AOS**
- **Physikingenieurwesen PI**
- **Physical Engineering PI-AOS**
- **Wirtschaftsingenieurwesen WI**

Prof. Dr.-Ing. Alexander Kern

Aachen University of Applied Sciences
Campus Jülich

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Basics

Problem 1.1

The kinetic energy of a body with mass *m*, moving with velocity *v* , is given by

$$W_{kin} = \frac{1}{2} \cdot m \cdot v^2 .$$

Modify this equation, so that the units of energy results in kWh, if the mass is expressed in tons and the velocity in km/h.

Problem 1.2

Determine an equation which relates the weight (in Newtons) of 100 m copper wire to its diameter (in mm).

Values: $\rho_{Cu} = 8{,}93 \frac{g}{cm^3}$; $g = 9{,}81 \frac{m}{s^2}$

Problem 1.3

At time $t = 0$ a vehicle with an initial velocity $v_0 = 54$ km/h decelerates uniformly with a deceleration rate of 5 m/s^2. Derive related dimensional equations, where the time *t* is expressed in seconds:

a) for the velocity *v* in km/h;
b) for the distance *x* in meters covered since the time $t = 0$.

Problem 1.4

The electric conductivity of copper is $\kappa_{Cu} = 56 \frac{S \cdot m}{mm^2}$.

a) Find the corresponding value in units of $\frac{S}{cm}$.
b) Find the electric resistivity ρ_{Cu} in $\mu\Omega * m$.
c) What is the length of a copper cable having a resistance $R = 1\ \Omega$, if its cross-sectional area is $A = 1{,}5$ cm^2?
d) What is the cross-sectional area of an aluminium cable having the same length and the same resistance?

   (electric conductivity of aluminium is $\kappa_{Al} = 35 \frac{S \cdot m}{mm^2}$)
e) Derive an equation which relates the electric resistance *R* (in mΩ) of a copper conductor to its cross-sectional *A* (in cm^2) and to its length *l* (in km).
f) Derive an equation which relates the electric conductance *G* (in S) of an aluminium conductor to its cross-sectional *A* (in cm^2) and to its length *l* (in km).

Analysis of DC circuits

Problem 2.1

A resistive strain gauge of resistance R = 600 Ω has an active length of 2,5 cm. It is made of 16 longitudinal wires (in series) having diameter d = 0,0205 mm.

a) Determine the resistivity of the wire material.
b) What voltage is there at the strain gauge at a current of 15 mA?

Problem 2.2

In order to measure strong currents, a resistor in the form of a hollow cylinder is to be made of manganin. At current 1 kA, the voltage across the resistance should be 200 mV. The outside diameter of the hollow cylinder be d_a = 5,5 cm and its length l = 12 cm. The resistivity of manganin is ρ = 0,42 Ωmm^2/m.

a) Determine the conductance of the resistor.
b) How thick must the wall of the hollow cylinder be?

Problem 2.3

The weight of a railway track related to its length is 436 N/m. The density of iron is ρ = 7,8 g/cm^3, its conductivity κ = 10,53 Sm/mm^2. For the acceleration of gravity, use the value g = 9,81 m/s^2. What is the length of the rail if its resistance is R = 1 Ω?

Problem 2.4

A cylindrical coil having inside diameter d_i = 1,5 cm and outside diameter d_a = 9,5 cm consists of n = 100 layers of varnished copper wire, each layer having m = 125 turns. Without insulation, the wire's diameter is δ = 0,3 mm.

a) What is the resistance of the coil at temperature 20℃?
b) What is the voltage across the coil if the current is 200 mA?
c) As the coil warmes up the current decreases to 160 mA at constant voltage. What is the average temperature of the coil now?

Use the values: κ = 56 Sm/mm^2 ; α_{20} = 3,93·10^{-3} K^{-1}

Problem 2.5

The tungsten filament of an electric light bulb of power P = 100 W has a temperature ϑ = 2500 °C when being operated at voltage U = 230 V.

a) Determine the resistance of the filament at operating temperature.
b) What is the resistance of the filament at room temperature (20 °C)?
c) Determine what the current and power would be, if the filament did not heat up while being operated.

α_{20} = 4,1·10^{-3} K^{-1} β_{20} = 10^{-6} K^{-2}

Problem 2.6

An electric heater has power 1 kW at voltage 230 V. Its heating wire has a diameter of 0,6 mm and is made of a chromium-nickel alloy having the resistivity ρ_{20} = 1,13 Ωmm^2/m and a temperature coefficient of α_{20} = 0,2·10^{-3} K^{-1}. In rated operation the temperature of the heating wire increases by $\Delta\vartheta$ = 500K. What is the length of the heating wire?

Problem 2.7

A voltage dependent resistor (VDR) is being used as a part of an overvoltage arrester in a medium high voltage system of U = 15 kV. Its characteristic can adequately be described by the following equation:

$$\frac{I}{A} = \left(\frac{U}{9kV}\right)^4$$

a) Determine the current through the voltage dependent resistor at 15 kV.
b) What current flows through the VDR if the voltage is increased by 20 %?
c) Derive an equation which relates the electric resistance R (in kΩ) to the voltage U (in kV).

Problem 2.8

Voltage and current have been measured at the terminals of a linear electric source with two different loads. These are the results:

Load 1 U_1 = 240 V; I_1 = 10 A
Load 2 U_2 = 225 V; I_2 = 40 A

Sketch: a) the equivalent circuit for this voltage source
b) the U/I - characteristic
Determine: c) the intrinsic resistance
d) the open-circuit voltage
e) the short-circuit current.

Problem 2.9

A linear electrical source has an open-circuit voltage of 10 V. If a resistance of 40 Ω is connected to the source, the terminal voltage decreases to 8 V.

Find:
a) the intrinsic resistance
b) the short-circuit current
c) the equivalent voltage-source circuit
d) the equivalent current-source circuit.

Problem 2.10

A resistance load of R_a = 24 Ω is added to a linear electric source with an open-circuit voltage U_0 = 100 V and an intrinsic resistance R_i =1 Ω. For the following quantities, determine first the general formulas and then the numerical values.

Find:
a) the current I
b) the terminal voltage U
c) the power P_a dissipated by the resistance R_a
d) the power P_i dissipated by the internal resistance, if the equivalent voltage source circuit is understood to be an accurate description of the processes going inside the source as well
e) the efficiency η in question d)
f) the resistance load $R_{a/max}$, *for which* the source provides the greatest possible power $P_{a/max}$
g) the power $P_{a/max}$ and the efficiency η in question f).

Problem 2.11

The heater winding of an immersion coil has a resistance of 84,8 Ω. The device is immersed into a container with 1 kg of water at a temperature of 21,8 ℃. After the coil was attached to a voltage of 230 V for 7 min, the water has a temperature of 80℃. The specific heat of water has the value $c = 1{,}163 \cdot 10^{-3} \frac{kWh}{kg \cdot K}$.

Determine the thermal efficiency of the device.

Problem 2.12

An electric stove with a nominal voltage of U = 230 V has two heating coils and four power levels, three of them being 1250 W, 750 W and 500 W. The fourth level is lower than those.

a) How will the two coils be connected in order to obtain the four levels?
b) Which resistances do the two coils have?
c) What is the lowest power level?

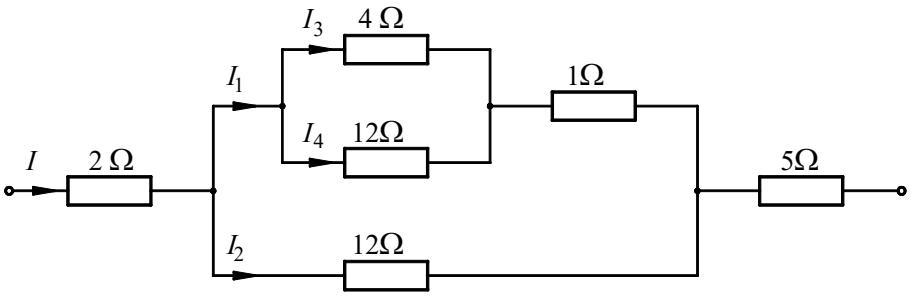
Problem 2.13

A light bulb with the value "6V, 3W" and a light bulb with the value "6V, 2W" will be connected in series to a 12V-power supply.

Is this wiring recommended? What will happen? (Explain your answer!)

Problem 2.14

The current I = 8 A is flowing through the circuit shown below.

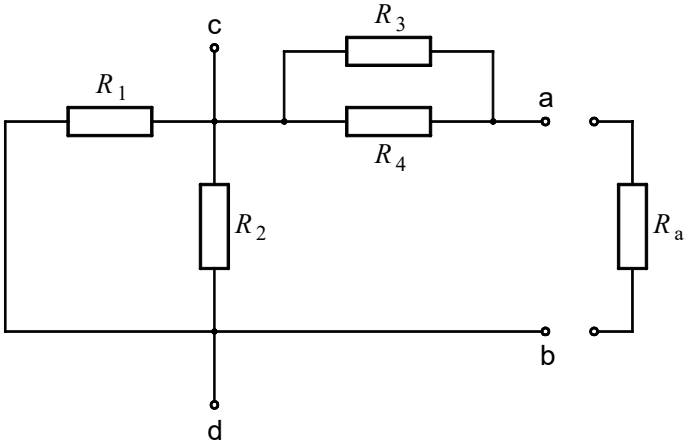


a) Determine the currents I_1, I_2, I_3, and I_4 .
b) Determine the power dissipated by the individual resistances and the total circuit.

Problem 2.15

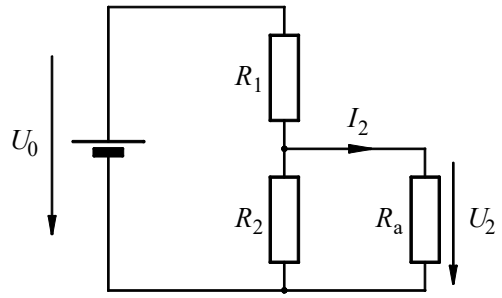
The resistances in the network shown below have the following values:

R_1 = 10 Ω
R_2 = 40 Ω
R_3 = 5 Ω
R_4 = 15 Ω
R_a = 8.25 Ω.



a) What is the resistance between the the terminals **a** and **b**?
b) What is the resistance between the terminals **c** and **d**, if terminals **a** and **b** are open?
c) What is the resistance between the terminals **c** and **d**, if at terminals **a** and **b** the resistance R_a is connected?

Problem 2.16



The illustration describes a voltage divider with a total resistance $R_1 + R_2$ = 1 kΩ. The setting is such, that R_1 = 600 Ω and R_2 = 400 Ω.

Further values: U_0 = 60 V und R_a = 960 Ω.

a) Determine current I_2 and voltage U_2.
b) Determine the power dissipated by the resistance R_a.
c) What are the values of R_1 and R_2 so that half the voltage of U_0 is across R_a?

Problem 2.17

A battery has an open-circuit voltage of 60 V and an intrinsic resistance of 2 Ω. This battery is charged by a voltage source with an open-circuit voltage of 110 V and an intrinsic resistance of 1 Ω .

a) What is the charging current?
b) What is the voltage at the terminals during charging?

Problem 2.18

A battery for a car has an open-circuit voltage of U_{0B} = 12 V. The car's starter takes on a current of 175 A. This load decreases the voltage at the battery's terminals to 7,8 V.

a) Determine the intrinsic resistance of the battery.
b) The starter takes on a power of 1 kW. What is the cross-sectional area of the 752 mm long copper cable (κ_{Cu} = 56 Sm/mm^2) connecting battery and starter, if the resistance of the return circuit (car body) and the starter switch together amount to 10 mΩ?
c) The car's dynamo has the open-circuit voltage U_{0D} = 13,5 V. The intrinsic resistance of the dynamo including the battery leads is 96 mΩ. What is the charging current of the battery?
d) Determine the voltage at the battery's terminals during charging.
e) Determine the power at the battery's terminals during charging.
f) What is the maximum power one can expect from the battery? Give the values of voltage and current at battery's terminals when being operated this way.

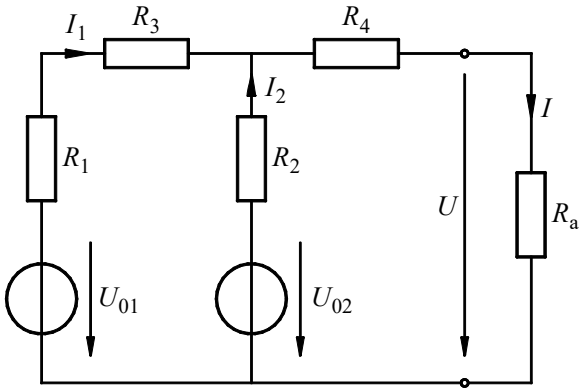
Problem 2.19

The elements in the circuit have the following values:

U_{01} = 220 V U_{02} = 200 V

R_1 = 4 Ω R_2 = 10 Ω
R_3 = 6 Ω R_4 = 5 Ω
R_a = 200 Ω

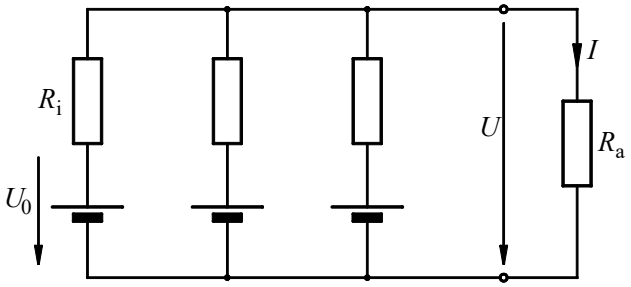
Calculate the currents I_1, I_2, I and the voltage U.



Problem 2.20

Three batteries with values the U_0 = 6 V and R_i = 0,3 Ω are connected in parallel.

a) Determine current and voltage at the terminals for R_a = 0,1 Ω .
b) How will these values change, if one of the batteries has the values U_{01} = 4,5 V and R_{i1} = 0,5 Ω?

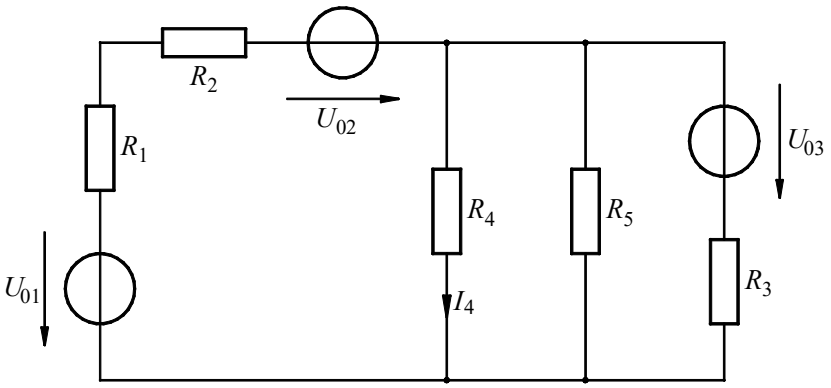


Problem 2.21

The elements in the circuit have the following values:

U_{01} = 35 V
U_{02} = 20 V
U_{03} = 24 V

R_1 = 2 Ω
R_2 = 3 Ω
R_3 = 4 Ω
R_4 = 34 Ω
R_5 = 20 Ω.



Determine the current I_4 using the equivalent voltage source method.

Problem 2.22

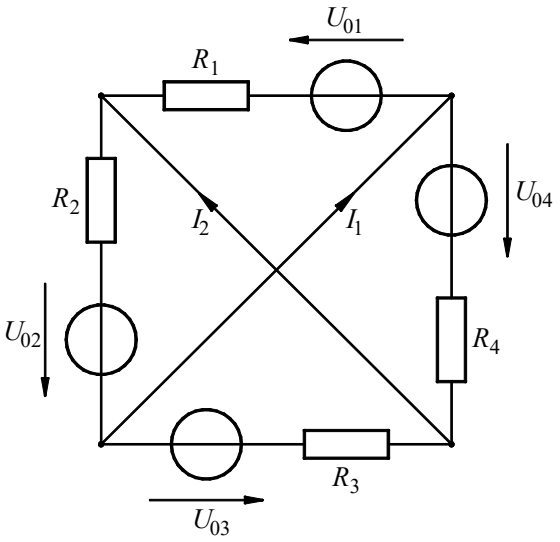
Calculate the currents I_1 and I_2.

U_{01} = 4 V R_1 = 2 Ω

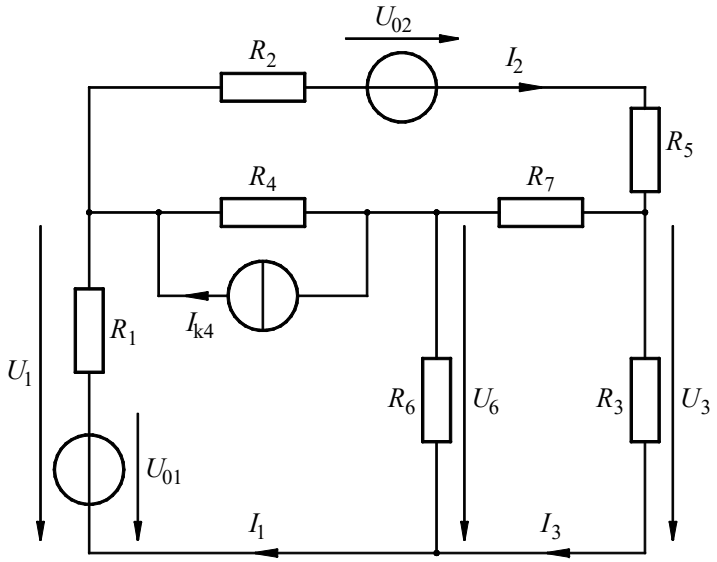
U_{02} = -4 V R_2 = 2 Ω

U_{03} = 8 V R_3 = 2 Ω

U_{04} = 4 V R_4 = 2 Ω



Problem 2.23



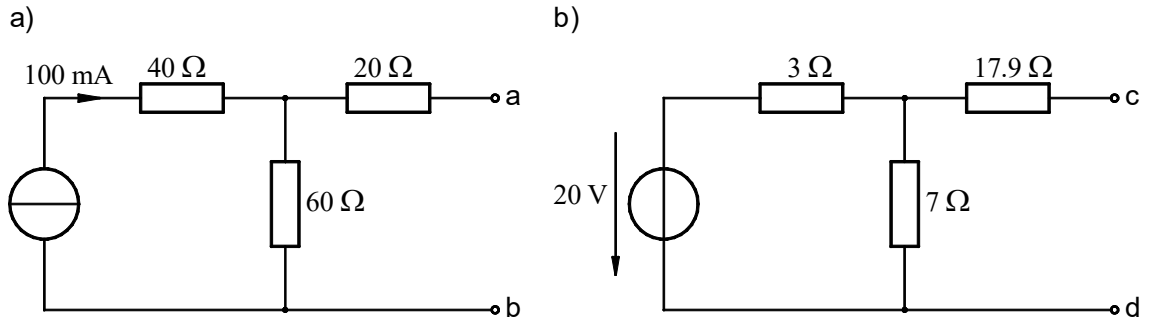
R_1 = 20 Ω
R_2 = 13 Ω
R_3 = 30 Ω
R_4 = 30 Ω
R_5 = 17 Ω
R_6 = 10 Ω
R_7 = 20 Ω

U_{01} = 90 V
U_{02} = 147 V

I_{k4} = 1,1 A

a) Use the mesh current analysis to determine the currents I_1, I_2, and I_3.
b) Use the node voltage analysis to determine die voltages U_1, U_3, and U_6.

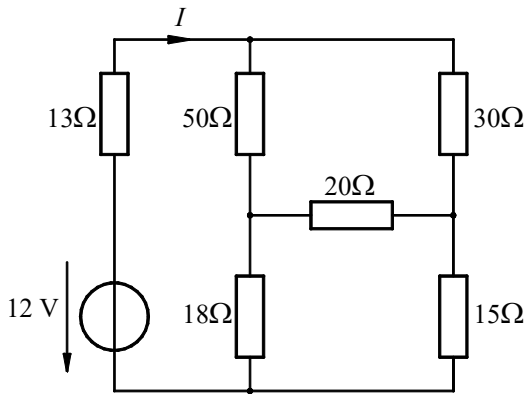
Problem 2.24



a) Determine in circuit a) the equivalent voltage source for the terminals **a** and **b**.
b) Determine in circuit b) the equivalent current source for the terminals **c** and **d**.

Problem 2.25

Calculate current I in the illustration.



Problem 2.26

In a linear electric network a voltage will be measured by means of a multimeter with 5 kΩ/V. In the 20V- range the instrument indicates 8 V, in the 10 V- range only 6 V. The inaccuracy of the measuring device can be ignored.

a) What was the voltage before the multimeter was connected to the network?
b) What does the instrument show in the 5 V- and 2 V- range?

Problem 2.27

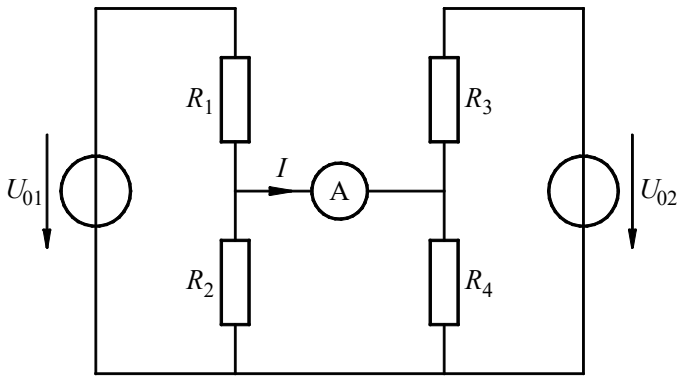
Using an ammeter with 1 mA full range and an intrinsic resistance of 100 Ω, a multimeter can be constructed with the staggered scale set to 1, 2, 5, 10, ...

a) Determine the smallest possible voltage range.
b) Which dropping resistors will be needed, if the highest voltage range should be 500 V and the particular resistor for any range is connected in series to those of the lower ranges?
c) Which shunt resistor will be needed for measuring currents up to 10 A?
d) What is the maximum power dissipated in the series resistor and in the shunt?

Problem 2.28

U_{01} = 40 V
U_{02} = 20 V

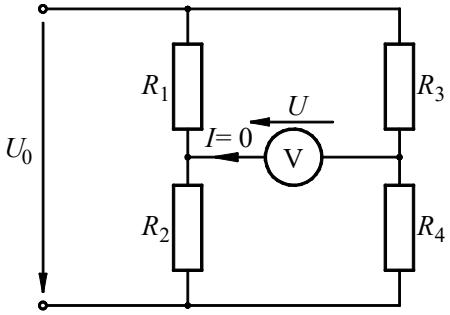
R_2 = 40 Ω
R_3 = 100 Ω
R_4 = 100 Ω



Which value must the resistance R_1 have in the compensation circuit shown, so that I = 0?

Problem 2.29

A bridge-circuit of strain gauges as shown below is being used to measure the strain of objects being bent.
The applied voltage is U_0 = 20 V. The resistance amounts to $R_1 = R_2 = R_3 = R_4$ = 600 Ω.
The voltmeter's resistance is very high, so it can be assumed that I = 0.

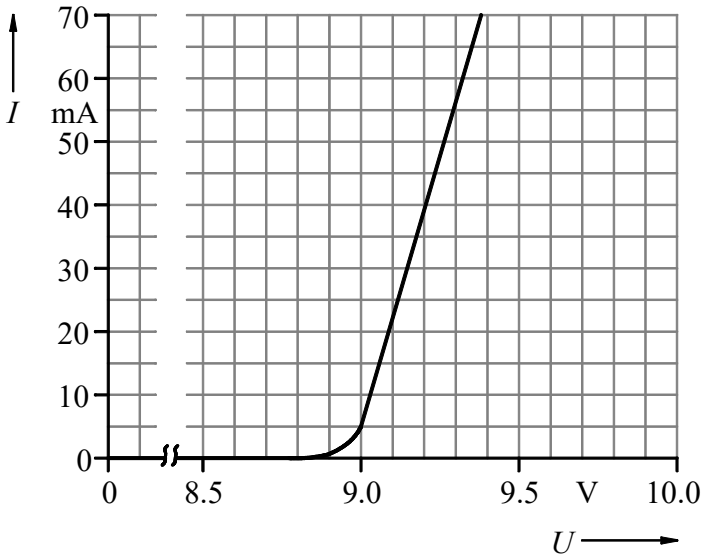


a) Determine the voltage U, if strain gauge 1 is being stretched by 0,05% and so its resistance R_1 increases by 0,1%
b) Determine the voltage U, if the resistances R_1 and R_4 increase by 0,1%.
c) Determine the voltage U, if the resistances R_1 and R_4 increase by 0,1% and the resistances R_2 and R_3 decrease by 0,1%.

Problem 2.30

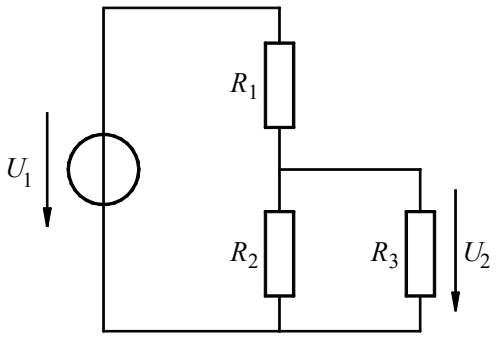
A Z-diode is connected to a linear electric source having the open-circuit voltage U_0 = 14 V.

The I/U- characteristic of the Z-diode is shown in the diagram.



a) Determine the current, if the voltage at the Z-diode has the value U= 9,25 V.
b) Determine the internal resistance R_i of the source.
c) By what percent will the voltage at the Z-diode change if the open-circuit voltage of the source (by a constant internal resistance R_i) drops from 14 V to 12 V (i.e. by about 14%)?

Problem 2.31



U_1 = 12.15 V
R_1 = 45 Ω
R_2 = 180 Ω

a) Determine the voltage U_2, if the resistance $R_3 = \infty$.
b) Determine the voltage U_2, if the resistance R_3 = 360 Ω.
c) What values result in the cases a) and b), if the resistor R_2 is replaced by a Z-diode (I/U-characteristic see diagram for problem 2.30)?

Electrostatic Field

Problem 3.1

The distance of two point charges Q_1 and Q_2 in air ($\varepsilon = \varepsilon_0 = 8{,}85 \cdot 10^{-12}$ As/Vm) is d = 0,5 cm. Determine the value and the direction of the force between the two charges, if Q_1 = 1 nC and:

a) $Q_2 = 0$ b) $Q_2 = -2$ nC c) $Q_2 = 4$ nC

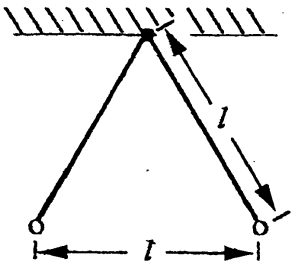
Problem 3.2

A small sphere carrying a charge of 1 nC is hanging from a thread between two large vertically arranged even electrodes with 5 cm distance.

a) Which force is being exerted on the sphere, if a voltage of 5 kV is applied between the two electrodes?
b) What kind of movement will the sphere carry out after the voltage is switched on?

Problem 3.3

Two identically charged small spheres hanging in air can be regarded as point charges. They have a mass m = 8,93 mg and repel each other as shown opposite. The length of the holding thread is l = 8 cm and the distance between the spheres as well.



Calculate the charge of the spheres.

Values: $g = 9{,}81$ m/s^2
$\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ As/Vm

Problem 3.4

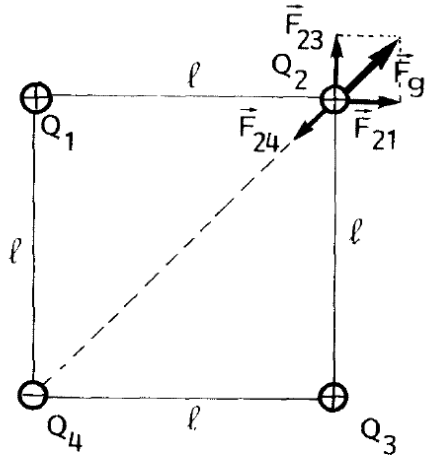
The Bohr radius (radius of the inner electron orbit within a hydogen atom) is $r = 53 \cdot 10^{-12}$ m. Give the electric field strength at this orbit.

Value: $e = 0{,}16 \cdot 10^{-18}$ As

Problem 3.5

Four equal charges are placed in the corners of a square with the side length *l*. The whole arrangement is in vacuum. Q_1, Q_2 and Q_3 are positive charges, Q_4 is negative.

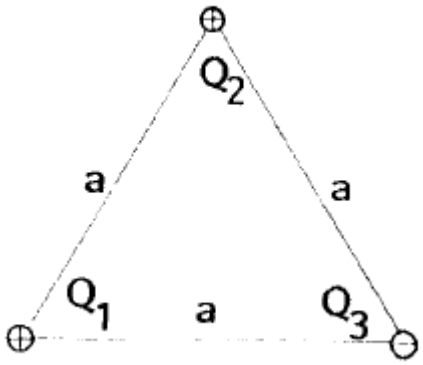
Determine the resulting force on the charge Q_2.



Problem 3.6

Three point charges Q_1 = 2,5•10^{-8}As, Q_2 = 1,5•10^{-8}As and Q_3 = -2•10^{-8}As are placed in the corners of an equilateral triangle with the side a = 10 cm, like shown opposite. The arrangement is in air (ε_0 = 8,85•10^{-12} As/Vm).

Define the origin of a cartesian 2D coordinate system in the center point of the triangle and calculate:



a) the absolute values of the three electric field strengths, which are originated by the three charges, in the center point;
b) the absolute value of the resulting electric field strength in the center point;
c) the angle α of the resulting electric field strength with the horizontal x-Axis.

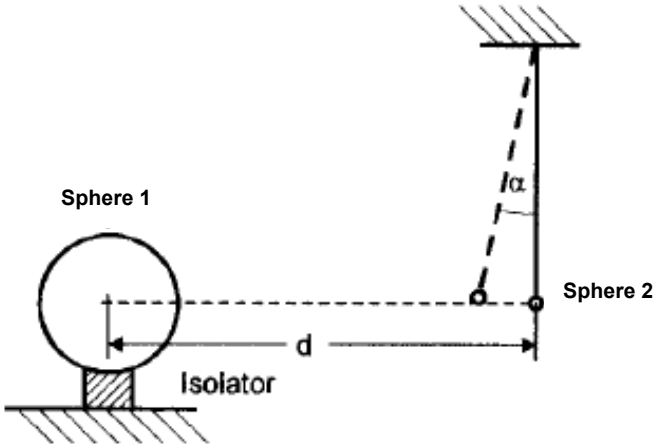
Problem 3.7

Two equal charges are located in an cartesian coordinate system (x,y) in the points (-1,0) and (1,0). Determine the direction of the common electric field vectors originated by both charges in the points (1,2), (3,0), (3,3) and (4,1). How are the electric field lines orientated in farer distances?

Problem 3.8

Opposite to an isolated sphere 1 a second sphere 2 is hanging freely movable at a distance d = 1 m. The capacitance between the two spheres is C_{12} = 2 pF. Between the two spheres a voltage of U_{12} =50kV is applied. The arrangement is in vacuum.

a) What is the electric field strength originated by sphere 1 at the position of sphere 2?
b) What is the attraction force F_{12} between the two spheres?
c) Due to the attraction force the hanging of sphere 2 is deflected by an angle α. What is the angle α, if sphere 2 has a mass of m = 10^{-3} kg? (the deflection of the sphere is assumed to be so small, that the variation of the electric field strength can be neglected).



Problem 3.9

The charge density ρ is given as a function of the distance r from the center point:

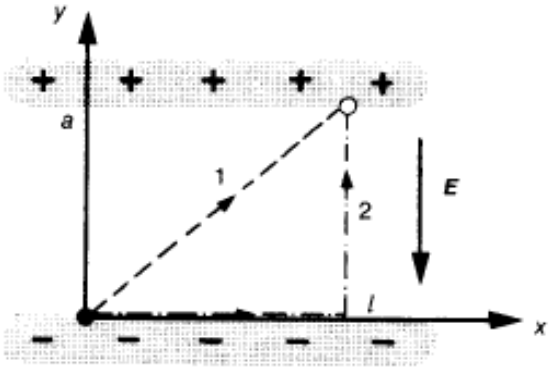
$$\frac{\rho}{\frac{pAs}{cm^3}} = \frac{3}{1+\left(\frac{r}{cm}\right)^3}$$

Determine the geometric distributions of the electric flux density $D(r)$ and of the electric field strength $E(r)$ related to the polar coordinate r, if the relative permittivity ε_r = 1 (ε_0 = 8,85·10^{-12} As/Vm).

Problem 3.10

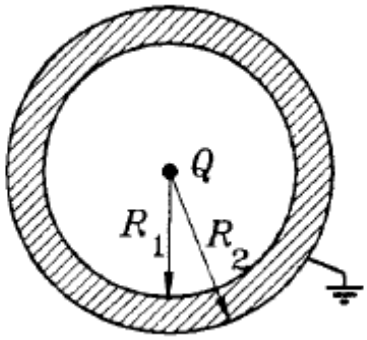
The electric field strength within the given two plate arrangement is homogeneous and described by:

$$\boldsymbol{E}_0 = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} 0 \\ -10 \\ 0 \end{pmatrix} \text{ kV/cm.}$$



Determine the work which has to be carried out to move a charge Q = 1 µC from point P_1 *(0, 0, 0)* to point P_2 *(l, a, 0).* What is the voltage between the two plates?

Problem 3.11



A positive point charge Q is located concentrically in the center of a metal hollow sphere (inner radius R_1, outer radius R_2).

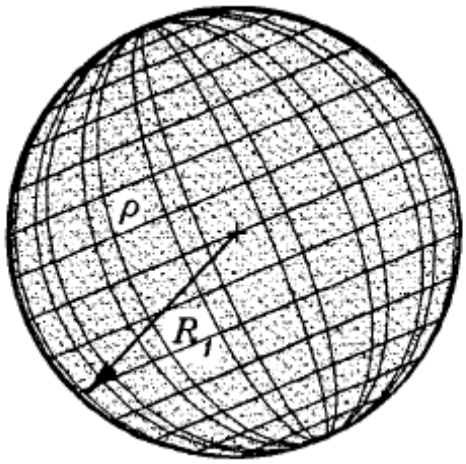
a) The metal hollow sphere is earthed; its potential is zero.
Calculate the geometrical distribution of the electric field strength E and of the electric potential φ in the whole area ($0 < r < \infty$). Draw the geometrical distributions $E = f(r)$ and $\varphi = f(r)$.

b) The earthing of the metal hollow sphere is now replaced. What are the changes? Give also for this case the geometrical distributions $E = f(r)$ and $\varphi = f(r)$.

c) As a difference to part a) the earthing of the metal hollow sphere does not exist from the beginning. Calculate the geometrical distribution of the electric field strength E and of the electric potential φ for this case. Draw again the geometrical distributions $E = f(r)$ and $\varphi = f(r)$. Use: potential $\varphi(r \rightarrow \infty) = 0$.

Problem 3.12

A space charge is spherically and homogeneously distributed (radius R_1) and contains a total charge Q. In the whole space we have: $\varepsilon = \varepsilon_0$.



a) What is the charge density ρ of the space charge? Write $\rho = f(R, R_1, Q)$, so that only the variable R and the given quantities R_1 and Q are contained.

b) Determine the geometrical distribution of the electric field strength $E = f(R)$ in the whole space (0 < R < oo). Draw this distribution.

c) Determine and draw also the geometrical distribution of the electric potential $\varphi = f(R)$ in the whole space (0 < R < oo).

Remark: φ(R -> oo) = 0

Problem 3.13

The electrodes of a plate capacitor are at a distance of 0,2 mm. Between the electrodes a voltage of 100 V is applied. The relative permittivity is $\varepsilon_r = 2$.

a) What is the electric field strength between the electrodes?
b) What is the electric flux density?
c) How much charge can be found on every cm^2 of the plates' surface?
d) What is the capacitance related to the plates' surface?

Problem 3.14

A coaxial cable (inner and outer conductor are coaxial cylinders) has an inner conductor of 1 cm diameter and an outer conductor of 2,6 cm diameter. The cable is operated at a voltage of 8,6 kV.

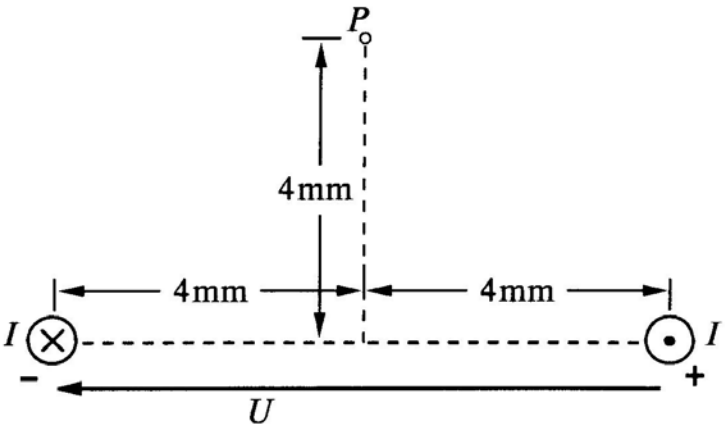
a) Determine the electric field strength at the inner conductor.
b) Determine the electric field strength at the outer conductor.

Problem 3.15

Two 100 m long copper wires with a cross-sectional area of 4 mm^2 are laid in parallel in air with a distance of 1 m. Due to an existing voltage U the first wire contains the electric charge Q = +1,25 μC and the second wire $-Q$ = -1,25 μC.

a) Determine the electric flux density exactly in the middle between the two wires.
b) What is the electric field strength at this position? Give the direction.
c) What is the force the two wires generate on each other? Do the two wires attract or repel each other?

Problem 3.16



The capacitance of a straight, long two-wire line, related to its length, is

$$\frac{C}{l} = 5\frac{pF}{m}.$$

The two wires of the line have a distance of 8 mm. The line connects a DC voltage source with a consumer. Like shown above, the current *I* in the left wire flows into the drawing plane and in the right wire out of the drawing plane. The left wire has negative, the right wire positive potential. Between the two wires at the shown location there is a voltage *U* = 30 V. The relative permittivity is 1.

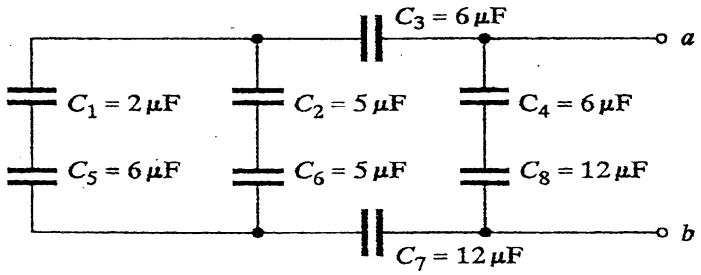
a) Is the voltage source in front of or behind the drawing plane?
b) What is the stored electric charge related to the wire's length *Q/l*?
c) Calculate the electric flux density *D* and the electric field strength *E* for the point *P*.
d) Plot the vector's directions $\vec{D}$ and $\vec{E}$ in the point *P* in the given drawing.

Problem 3.17

A spherical capacitor consists of two concentric spheres. The outer sphere has a radius *r* = 80 cm.

a) How large must be the radius r_i of the inner sphere so that, for a constant voltage between the two spheres, the electric field strength at the inner sphere $E(r_i)$ has the minimum possible value?
b) Compared to case a), by which factor will the electric field strength at the inner sphere increase, if its radius is set to r_i = 20 cm and the voltage is constant?
c) Compared to case a), by which factor will the electric field strength at the inner sphere increase, if its radius is set to r_i = 20 cm and the charge is constant?
d) Give the ratio of the capacitances in case a) compared to cases b) and c).

Problem 3.18



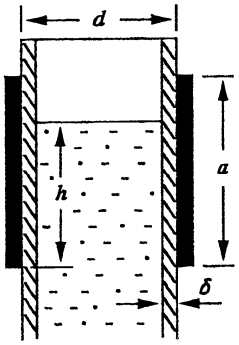
Determine the voltages at the capacitors C_1 … C_8, if a voltage of 12 V is applied between the terminals *a* and *b*.

Problem 3.19

A tubular capacitor with the dimensions l = 10 cm, r_i = 0,5 cm, r_a = 1 cm is filled with oil (ε_r = 2). The capacitor is charged to a voltage of 10 V and then removed from the voltage source. Stray fields are to be ignored.

a) Determine the capacitance of the capacitor.
b) How much energy is stored in the capacitor?
c) The inner cylinder is half-removed on its axis out of the outer cylinder. How much energy is stored in the capacitor now?
d) Where does the energy difference come from?
e) What voltage can be measured at the terminals of the capacitor in case c)?

Problem 3.20



In a filling-level measuring device as shown opposite, a receptacle is enclosed by two flat square electrodes with side lengths a = 10 cm and a distance d = 2 cm. The walls of the receptacle have a thickness δ = 2 mm and are made of a material with the relative permittivity ε_{r1} = 2. Stray fields are to be ignored.

The electric field constant is: $\varepsilon_0 = 8{,}85 * 10^{-12}$ As/Vm

a) Determine the capacitance of the device, if the receptacle is empty.
b) Determine the capacitance of the device, if the receptacle is completely filled with water (ε_{r2} = 80).
c) Determine a relationship between the filling level h and the capacitance C.

Problem 3.21

The plates of a plate capacitor have a distance d = 1 mm. A voltage of U = 1 kV is applied.

a) What are the pressure exerted on the electrodes and the energy density, if there is oil (ε_r = 2) between the plates?
b) What are the pressure and the energy density, if the capacitor is charged as an air capacitor, then disconnected from the source and finally filled with oil?

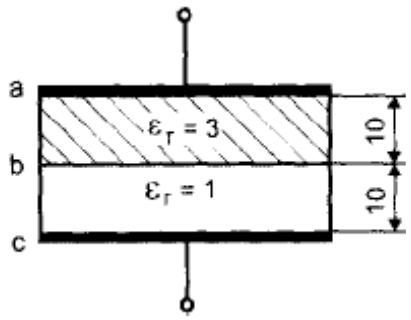
Problem 3.22

A spherical electrode is located in the center of a hollow sphere with a radius r_a = 5 m The hollow sphere has a conductive inner surface. Due to good conditions the dielectric strength of the air in the hollow sphere has a value of E = 30 kV/cm.

a) Within which limits must be the radius r_i of the spherical electrode, if a voltage of U = 712,5 kV is applied between the spherical electrode and the hollow sphere, and the given dielectric strength of air may not be exceeded at any position?
b) What is the capacitance of the arrangement, if r_i = 25 cm?
c) What is the charge on the electrodes in this case and for U = 712,5 kV?
d) How much energy is stored?
e) What is the maximum value of the energy density? Where does it occur?

Problem 3.23

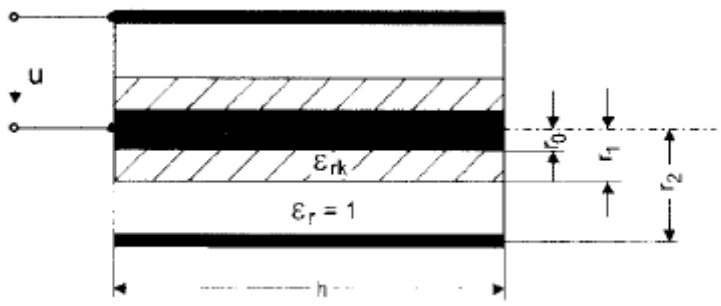
A plate capacitor consists of three thin metal plates with an area A = 100 cm^2. The plate capacitor is charged up to a voltage U = 1000 V and then removed from the voltage source.



a) What is the whole capacitance of the arrangement? What is the voltage U_{ab} between the plates *a* and *b*?
b) The distance between the plates *b* and *c* is increased to 20 mm by moving the plate *c*. What are the voltages U_{ab}, U_{bc} and U_{ac} now?
c) Calculate the energy of the whole arrangement before and after the plate *c* is moved. Explain a possibly occurring difference of the energies.
d) After moving the plate *c*, a 10 mm thick layer is brought between the plates *b* and *c*. What is the relative permittivity ε_r of the layer, so that the voltage U_{ac} shows the value 1150 V?

Problem 3.24

The inner metal electrode of a cylindrical arrangement is covered concentrically with a plastic layer having the relative permittivity ε_{rk}. The space to the outer metal electrode is filled with air ($\varepsilon_r = 1$). A voltage U is applied to the arrangement.



For $r_1 = 1{,}6\ r_0$, $r_2 = 4\ r_0$ and $\varepsilon_{rk} = 3$, the following quantities are to be determined in a general description:

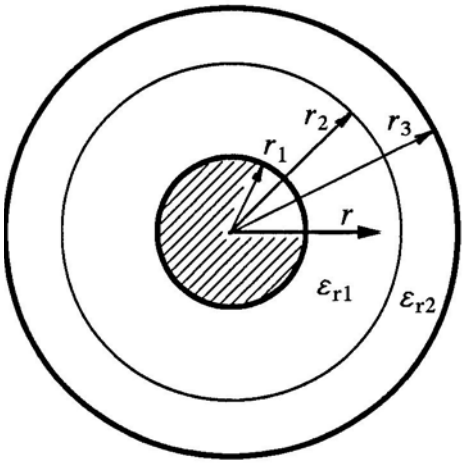
a) the whole capacitance C of the arrangement.
b) the geometrical distribution of the electric field strength in a normalized version $E(r/r_0)$.
c) for $U = 3219$ V and $r_0 = 5$ mm the geometrical distribution $E(r/r_0)$. Draw to scale this distribution.
d) the voltage U_{max} , so that the maximum tolerable electric field strength $E_{max} = 600$ V/mm is not exceeded at any position.

Problem 3.25

Between the two electrodes of the given spherical capacitor two different dielectric materials with the relative permittivities $\varepsilon_{r1} = 4{,}5$ and $\varepsilon_{r2} = 2{,}5$ are placed.

Dimensions:
$r_1 = 4$ cm; $r_2 = 9$ cm; $r_3 = 12$ cm.

The spherical capacitor is charged up to $Q = 5$ nC. At the boundary between the two dielectric materials there are **no** charges.



a) Describe the electric field strength E as a function of the geometrical coordinate r. Give related dimensional equations for the electric field strengths E in kV/m for r in cm.
b) What is the voltage between the inner electrode and the boundary layer? What is the voltage between the boundary layer and the outer electrode?

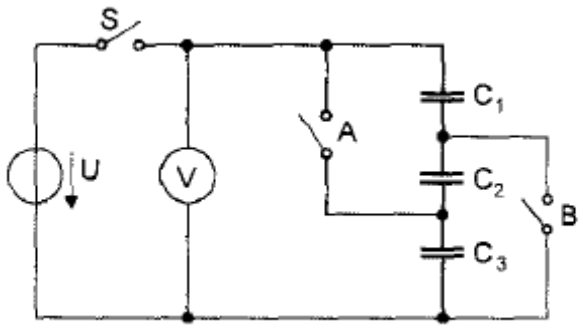
Problem 3.26

The capacitors in the given circuit are charged by the voltage source U = 275 V at open switches A and B. After the charging process is completed, the voltage source is disconnected by opening the switch S.

C_1 = 4 C C_2 =5 C C_3 = l0 C

What voltage values are shown at the voltmeter, if the following switches are closed:

a) switch A
b) additionally switch B.



Electrical Current Field

Problem 4.1

A plane area A = 100 cm^2 lies in a homogeneous electric current vector field. The surface vector $\vec{A}$ of the plane has the angle α to the current density vector $\vec{J}$. Determine the current I through the area A, if J = 2 A/cm^2 and:

a) α = 0° b) α = 60° c) α = 90°?

Problem 4.2

An inhomogeneous electric current field is described by $\vec{J} = \left(0,\ 0,\ \frac{x}{cm}\right)\frac{A}{cm^2}$.

What is the current through the square with the corner points:

P_1 = (2, 1, 0) cm ; P_2 = (4, 1, 0) cm ; P_3 = (4, 3, 0) cm ; P_4 = (2, 3, 0) cm?

Problem 4.3

An electric current field is described by $\vec{J} = \left(1,\ \frac{-x}{4cm},\ 0\right)\frac{A}{cm^2}$.

a) Sketch the vector $\vec{J}$ in the plane z = 0 for all points of a rectangular pattern, where the points have a distance of 1 cm from each other and for the range $|x| \le 4\,cm, |y| \le 2\,cm$.
b) Sketch the current field by the outline of the current lines.
c) Calculate the current through the square with the corner points P_1, P_2, P_3, P_4.
d) Calculate the current through the square with the corner points P_3, P_4, P_5, P_6.

Currents flowing in direction of the positive x- and y-axis should be considered positive. The coordinates of the points are:

P_1 = (4, -2, -2) cm ; P_2 = (4, -2, 2) cm ; P_3 = (4, 2, 2) cm ;
P_4 = (4, 2, -2) cm ; P_5 = (0, 2, -2) cm ; P_6 = (0, 2, 2) cm .

Problem 4.4

A two-dimensional potential field is described by: $\frac{\varphi}{kV} = -\ln\left\{\sqrt{\left(\frac{x}{cm}\right)^2 + \left(\frac{y}{cm}\right)^2}\right\}$

a) Sketch this potential field in the range 0,3 cm < r < 7,5 cm using $r = \sqrt{x^2 + y^2}$.
b) Determine the electric field $\vec{E}$ belonging to this potential field and sketch the electric field $\vec{E}$.
c) What is the voltage between the points P_1 = (2, 2) cm and P_2 = (5, 2) cm?

Problem 4.5

Calculate the electric field strength in a current carrying copper wire with a diameter of 2,76 mm for a current I = 20 A ($\kappa_{Cu} = 56 \frac{m}{\Omega \cdot mm^2}$).

Problem 4.6

A current of I = 10 μA flows through a hollow cylinder made of vulcanized fibre with $\rho = 10 \cdot 10^{12} \frac{\Omega \cdot mm^2}{m}$, the length l = 40 mm, the inner radius r_i = 10 mm, and the outer radius r_a = 20 mm from the inner to the outer surface. Calculate the current density and the electric field strength at the radius of 18 mm.

Problem 4.7

In a homogeneous electric field the electric field strength is E = 12 V/cm. The two points A und B are located on one electric field line in a distance of d = 5 cm. The potential of point B is higher than that of point A.

a) What are the potentials φ_A and φ_B of the points A and B, if the distance between point A and the reference-potential point C is a = 4 cm and the distance between points B and C is b = 3 cm?
b) Determine the three voltages U_{AB}, U_{BC} and U_{CA}?

Problem 4.8

The cross-sectional area of a steel-aluminium cable being used as an overhead line can be split up into an iron part of 8 mm^2 and an aluminium part of 48,3 mm^2. Through the cable flows a current of 170 A. The resistivities are ρ_{St} = 0,14 Ωmm^2/m for steel, and ρ_{Al} = 0,029 Ωmm^2/m for aluminium. What are the values of the electric field strengths and the current densities in aluminium and in steel?

Problem 4.9

For simplification the earth electrode of a power line tower is assumed to be a metal semi-sphere with a radius r_i = 7,5 m. The resistivity of the surrounding ground is ρ = 118 Ωm.

a) Determine the earthing resistance of the tower.
b) Determine the voltage distribution at the earth's surface in the vicinity of the tower, if it is struck by a lightning current of 5 kA.
c) At which distance from the tower's base will the potential drop by 4 kV, 8 kV, 12 kV?
d) Which voltage has the tower's base against a point with infinite distance from the tower?
e) What is the highest voltage between any two points at a distance of 1 m on the surface?

Problem 4.10

Calculate generally the resistance of a semi-circle shaped conductor with a square cross-section. Given are the inner radius r_i, and the outer radius r_a of the semi-circle, and the conductivity κ.

Problem 4.11

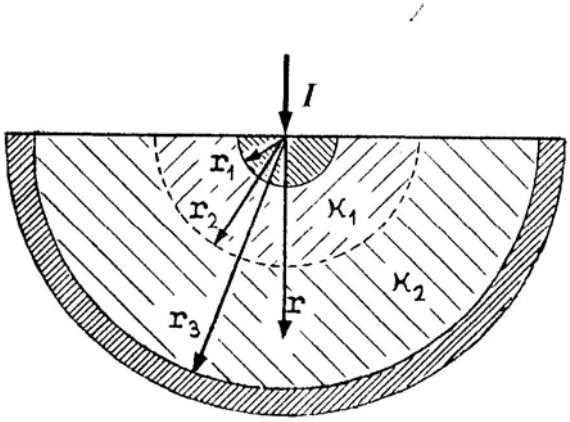
A cylindrical pipe made of laminated paper (resistivity ρ) has the outer diameter d, the wall thickness δ, and the length l.

a) Both face surfaces are covered with metal layers. What is the resistance between the two metal layers?
b) Now the inner and the outer surface are covered with metal layers (with the exception of both face surfaces). What is the resistance between the two metal layers now?
c) At which position do we find the maximum power density for case b)? Give the power density for a voltage U between the two metal layers.

All results shall only contain the given quantities ρ, d, δ, l and U.

Problem 4.12

The space between the two concentrical, semi-spherical electrodes is filled by two materials with different conductivities, like shown opposite. The current I flows from the inner to the outer electrode.



The following quantities are given: I, r_1, r_2, r_3, κ_1, κ_2

Calculate, using only the given quantities:

a) the current density J as a function of the geometrical coordinate r;
b) the electric field strength E as a function of the geometrical coordinate r;
c) the voltage U between the two electrodes;
d) the electric resistance R between the two electrodes.

Analysis of AC circuits

Problem 5.1

Given is a sinusoidal AC voltage with a maximum value $\hat{u}$ = 500 V and a frequency f = 50 Hz. At time t = 0 the instantaneous value is $u(t = 0)$ = 130 V (increasing).

a) Sketch the voltage vs. time graph.
b) Describe the voltage-time dependency with an equation.
c) Determine the zero phase angle φ?
d) At which time t_1 does the voltage first reach the maximum value?
e) At which time t_2 does the voltage first reach the value -400 V?

Problem 5.2

An AC voltage with the frequency f = 250 kHz is given. The positive and the negative half waves are symmetrical and may be obtained from each other by simply changing the sign. The positive half wave in the time period $-\mu s \leq t \leq 1\ \mu s$ is described by:

$$\frac{u}{V} = \left(\frac{t}{\mu s}\right)^2 - \left(\frac{t}{\mu s}\right)^4$$

Sketch the course of the voltage and calculate the form factor and the crest factor.

Problem 5.3

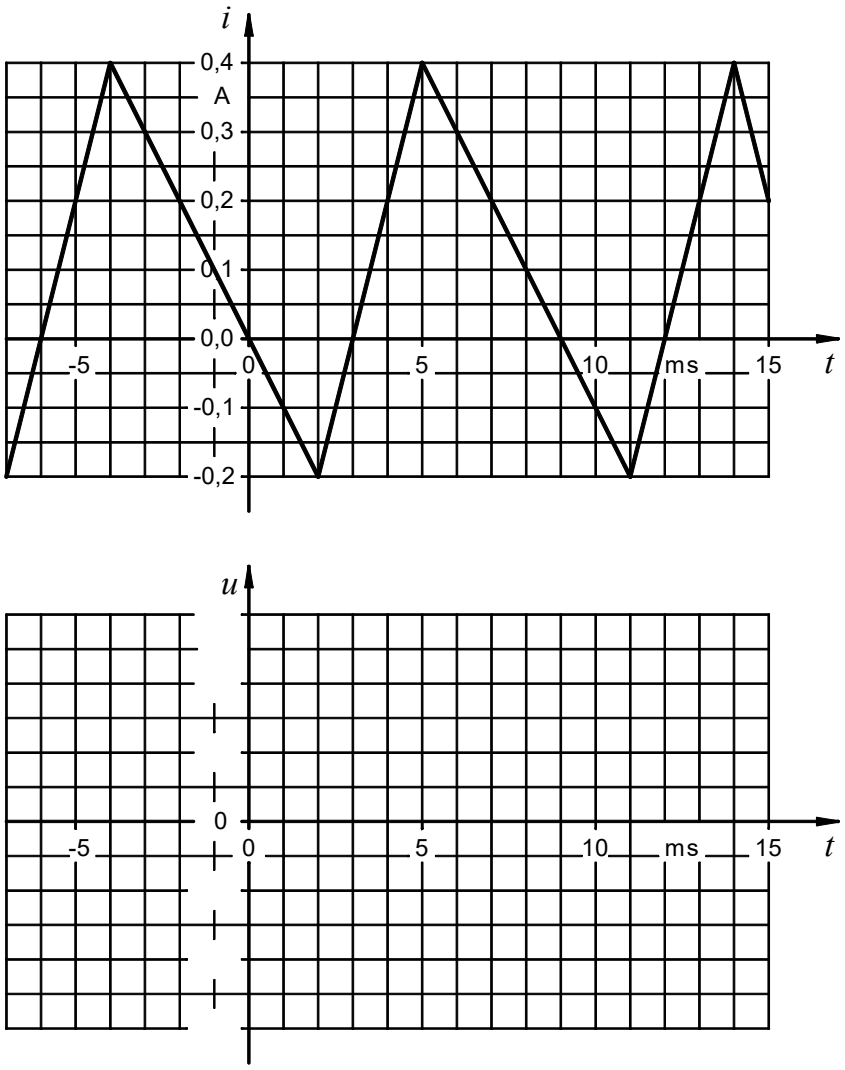
An AC voltage is described by the function $u = 8V \cdot \cos(\omega t) \cdot \sin(2\omega t)$, $\omega = 2000 \cdot \pi\, s^{-1}$.

a) Which frequencies are contained in this signal?
b) Determine the RMS value of each of the sinusoidal components.
c) Determine the RMS value of the entire signal.
d) Determine the rectified mean value of the signal.
e) Give the form factor of the signal.

Note: $\sin\alpha \cdot \sin\beta = \frac{1}{2}\{\cos(\alpha-\beta) - \cos(\alpha+\beta)\}$ $\quad$ $\sin\alpha \cdot \cos\beta = \frac{1}{2}\{\sin(\alpha-\beta) + \sin(\alpha+\beta)\}$

Problem 5.4

A current $i(t)$ with the time function given in the graph below is flowing through an inductance of L = 150 mH.



a) What is the frequency of the current?
b) Determine the direct component $i_=$ of the current.
c) Plot the time function of the voltage *u(t)* at the inductance into the lower diagram. The voltage axis has to be scaled before.
d) Calculate the RMS-value of the voltage determined under c).

Problem 5.5

A sinusoidal AC current with $\hat{\imath}$ = 350 mA and f = 50 Hz flows through the series connection of R = 100 Ω and L = 500 mH.

a) Determine the time dependency of the voltages $u_R(t)$ and $u_L(t)$.
b) Draw the functions $u_R(t)$ and $u_L(t)$ in the same time diagram and find graphically the course of the total voltage *u(t).*
c) Show the voltages $\hat{u}_R$ and $\hat{u}_L$ in a phasor representation of maximum values and find graphically the maximum value $\hat{u}$ and the phase position of the total voltage.
d) Check the results by calculation.

Problem 5.6

A sinusoidal a.c. voltage of frequency f = 50 Hz is applied to a loss-free capacitor. By means of two moving-iron instruments, a voltage of 220 V and a current of 11.75 mA have been measured.

a) Which breakdown voltage will the capacitor have, when it must be twice themaximum value of the voltage ?
b) Determine the magnitude of the capacitor's impedance and its capacitance.

Problem 5.7

The series equivalent circuit of a coil has the ohmic resistance R = 65 Ω and the inductance L = 500 mH. A sinusoidal a.c. voltage U= 221V of frequency f = 50 Hz is applied to the coil.

a) Give the impedance of the coil in the exponential form.
b) Determine the current and its phase shift versus voltage.

Problem 5.8

A coil is connected in series with a capacitor of capacitance C = 48 700 pF. When a sinusoidal a.c. voltage of U = 48.3 V, f = 2.4 kHz is applied, a current I = 200 mA flows through the circuit. The phase shift between current and voltage is $\varphi_i - \varphi_u = 40°$.

Determine the elements of the coil's series equivalent circuit if the capacitor may be regarded as being loss-free.

Problem 5.9

An a.c. current I = 8.8 A of frequency f = 50 Hz flows through the series connection of resistance R = 10 Ω, inductance L = 68.12 mH and capacitance C = 229 μF.

a) Determine the inductive reactance, the capacitive reactance, the entire reactance, and the impedance of the series circuit. Show your results in a true-to-scale phasor representation.
b) Determine magnitude of admittance, conductance, and susceptance of the circuit and outline the phasor representation of the admittance.
c) Calculate the voltages at the three elements of the circuit and the entire voltage. Outline the phasor representation of currents and voltages.

Problem 5.10

A series connection of a coil and a capacitor is given. The coil can be regarded as a resistance R = 500 mΩ and an inductance L = 1.5 :H in series. The capacitor is virtually loss-free and may be considered to be a pure capacitance C = 4.43 μF. At frequency f = 100 kHz there is a voltage of U_{sp} = 2 mV at the coil.

a) Determine the total voltage at the series connection.
b) Determine the phase shift of the total voltage compared to the voltage at the coil.

Problem 5.11

An a.c. voltage of U = 10 V with frequency f = 50 Hz is applied to a parallel connection of resistance R = 115.5 Σ and inductance L = 636 mH.

a) Determine the impedance of the circuit.
b) What current will flow?
c) Draw a true-to-scale phasor representation of currents and voltages.

Problem 5.12

An a.c. voltage of U = 300 mV with frequency f = 398 Hz is applied to the parallel connection of resistance R, capacitance C, and inductance L = 12 mH. The circuit takes a current of 10 mA.

Determine resistance R and capacitance C, if the voltage leads the current by the angle of $\varphi_i - \varphi_u$ = 41,4°.

Problem 5.13

In the range from 0 to 50 Hz the frequency response of a given coil can accurately be described by an RL - series connection. When a d.c. voltage of 220 V is applied, the coil takes a current of $I_{=}$ = l0 A; when applying an a.c. voltage of 220 V with frequency f = 50 Hz through the same coil only a current of I = 2 A flows. Determine for this coil:

a) the active resistance
b) the magnitude of impedance
c) the inductive reactance
d) the inductance in the series equivalent circuit
e) the phase shift of voltage against current
f) the voltages at resistance and inductance in the series equivalent circuit
g) the elements of the parallel equivalent circuit for frequency f = 50 Hz.
h) Outline for this coil the complete true-to-scale phasor representation for all currents and voltages in the series and the parallel equivalent circuits.

Problem 5.14

Inductance L = 69.7 mH , capacitance C = 0.503 :F, and resistance R are connected in series to the a.c. voltage *U*= 80 V. Determine for either R = 148.9 Ω and R = 34.79 Ω:

a) resonance frequency, characteristic impedance Z_0, and quality factor Q
b) bandwidth of the oscillating circuit
c) current at resonance frequency
d) voltages at inductance and capacitance when resonance occurs
e) frequencies at which the voltages at inductance and capacitance respectively reach their maximum values
f) maximum values of voltages at inductance and capacitance.

Problem 5.15

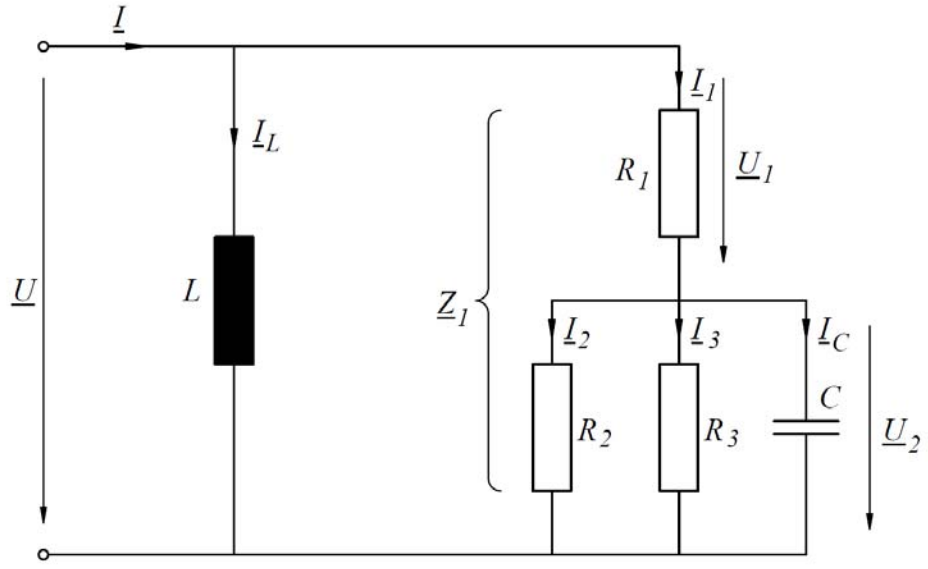
At constant voltage the following resonance for an oscillating circuit has been measured:

$\frac{f}{MHz}$	0.7	0.8	0.9	0.95	1.0	1.05	1.1	1.2	1.3	1.4
$\frac{I}{mA}$	3.18	4.87	8.25	10.68	12.0	10.78	8.68	5.75	4.23	3.36

a) Is this a series- or a parallel-resonant circuit?
b) Determine the quality factor.

Problem 5.16

Given is the following circuit:



U = 40 V R_1 = 6 Ω
f = 2.5 kHz R_2 = 6.3 Ω
C = 16.2 μF R_3 = 5.4 Ω

a) Determine the impedance Z_1 of that part of the circuit which contains the elements R_1, R_2, R_3, and C and give it in the exponential form.
b) Determine the voltages U_1, U_2 as well as the currents I_1, I_2, I_3, I_C giving magnitude and angle. The zero phase angle of the overall voltage U is $\varphi_u = 0$.
c) Calculate apparent power, active power, and reactive power at the impedance Z_1.
d) Which inductance L should be chosen to make the entire circuit an ohmic two-terminal?
e) What is the impedance of the entire circuit then?

Problem 5.17

A device for the electrical drying of wood (capacitor with the wood to be dried put between the plates) is supplied with an a.c. voltage of 4 kV at a frequency of 1.06 MHz. It has a power input of 6.25 kW at a current of 2.6 A.

a) Determine capacitance and resistance in the parallel equivalent circuit for the drying device.
b) What inductance must be connected in parallel to achieve resonance ?
c) Calculate the active current and the reactive current of the drying device and outline a complete phasor representation of currents and voltages.
d) What inductance must be connected in series with the drying device to compensate the reactive power completely ?
e) Determine for this case current and voltage of the drying device and calculate the power input.
f) Draw a complete phasor representation of currents and voltages for this case as well.

Problem 5.18

At a line voltage of U = 230 V (f = 50 Hz) a soldering iron has a rated power of P_N = 100 W. To avoid overheating during non-use, a series resistor reduces the voltage to 161 V.

a) Determine the resistance of the series resistor.
b) Determine the power which is absorbed by both the soldering iron and the series resistor.
c) The series resistor now is exchanged for a loss-free capacitor. What capacitance must this capacitor have if the voltage at the soldering iron shall continue to be 161 V ?
d) Determine active, reactive, and apparent power of the series connection of soldering iron and capacitor.

Problem 5.19

A fluorescent lamp operates on the mains (U = 230 V, f = 50 Hz). For a stabilized operating point, the fluorescent lamp will have a choke coil connected in series. Through the series connection flows a current I = 0.4 A. The lamp has power of 40W; the power dissipated in the coil is 10.6 W. At the lamp current and voltage are in phase.

a) Calculate the reactive power and the apparent power, as well as the power factor of the series connection.
b) Determine the inductance and the winding resistance of the choke coil.
c) Determine the working voltage at the lamp and the voltage at the coil.
d) In addition to the series connection of lamp and choke coil, a (loss-free) capacitor is now connected in series, whose capacitance is such that the circuit becomes ohmic-capacitive. Determine the capacitance so that the power factor continues to have the value found in a).
e) Determine the voltage at the capacitor.
f) A series connection of lamp and choke-coil (without capacitor) will be connected in parallel to a series connection of lamp, choke-coil and capacitor as described in d). Calculate the current of this duo-connection and give the power factor.
g) Which phase difference is there between the currents through the two fluorescent lamps?
h) Outline a complete phasor representation of currents and voltages.

Problem 5.20

A utility provider running three power stations is to provide an active power of 60 MW at power factor $\cos \varphi = 0.85$. Power station A provides an active power of 25 MW at $\cos \varphi = 0.9$; power station B provides an active power of 15 MW at $\cos \varphi = 0.8$; the rest is provided by power station C.

a) Determine reactive power and apparent power of power stations A and B.
b) What must be active power, reactive power, apparent power, and power factor of power station C?

Problem 5.21

An a.c. motor has mechanical performance of 20 kW at an efficiency of 85%; the power factor is $\cos \varphi = 0{,}75$. The motor is operating on an a.c. mains with $U = 230$ V and $f = 50$ Hz. To compensate the reactive power completely a capacitor is to be connected in parallel to the motor. Determine the capacitance of this capacitor.

Loci

Problem 6.1

An a.c. voltage of U = 9.4 V is applied to a parallel connection of resistance R = 235 Ω, inductance L = 6.8 μH, and capacitance C = 14.9 nF.

a) Calculate resonance frequency, characteristic impedance and quality factor ofthe oscillating circuit.
b) Determine the currents through the three components and the total current when resonance occurs.
c) Sketch the loci of admittance and impedance with frequency as a parameter.

Problem 6.2

A series resonant circuit contains components of inductance L = 41.38 mH, capacitance C, and resistance R_1. The characteristic impedance is Z_0 = 1.56 kΩ. The quality factor is $Q = \frac{12}{7}$.

a) Determine capacitance C and resistance R_1.
b) What is the resonance frequency f_0 ?
c) What are the cutoff frequencies f_{gu} and f_{go} ?
d) Calculate the impedance Z of the circuit for f_1 = 3.9 kHz and f_2 = 6.5 kHz.
e) Draw a graph of the locus for Z and outline the phasors of impedance for frequencies f_0, f_{gu}, f_{go}, f_1, f_2.
f) Draw a graph of the locus for Y. Indicate frequencies f_0, f_{gu}, f_{go}, f_1, f_2 as parameters of the locus.
g) How will the locus for Y change when a resistance R_2 = 2.73 kΩ is connected in parallel to the series resonant circuit?
h) Outline the locus of impedance for this circuit as well.

Three-phase systems

Problem 7.1

An unbalanced three phase load in delta connection operates on a balanced three phase mains (400 V, 50 Hz). Notice the following facts about the three phases:

load between L1 and L2:	the current is I_{12} = 5 A; power factor is cos φ = 0.8, capacitive.
load between L2 and L3:	the impedance, is $Z_{23} = R_{23}$ = 80 Ω (ohmic).
load between L3 und L1:	the apparent power is S_{31} = 3 kVA; reactive power is Q_{31} = 1,8 kvar (inductive).

a) Determine the total performance (active power) of the three phase load.
b) Determine phase currents I_{23} and I_{31}
c) What are the phase angles φ_{12}, φ_{23}, φ_{31} between voltage and current at each phase load?
d) Outline all phase voltages and phase currents in a true-to-scale phasor representation.
e) Determine the line currents I_1, I_2, I_3 .

Problem 7.2

A balanced three phase load in star connection operates on a balanced three phase mains (400 V, 50 Hz). It absorbs an active power of 12 kW at a power factor of cos φ = 0.866 (inductive).

a) Determine the current.
b) Determine the reactive power.
c) What values have resistance and inductance in the series equivalent circuit for each phase load ?
d) What values have resistance and inductance in the parallel equivalent circuit for each phase load ?
e) The reactive power is to be compensated by three capacitors connected in delta connection. Determine the capacitance of the capacitors.
f) What capacitance is required when connecting the capacitors in a star connection ?
g) Determine active power and reactive power of the three phase load when it is switched over from star connection into delta connection.

Problem 7.3

A balanced three phase load (P = 18 kW; cos φ = 0.6, inductive) operates on a balanced three phase mains (400 V, 50 Hz).

a) Determine active power, reactive power, and apparent power of each phase.
b) Determine phase voltage and phase current in case the load is in delta connection.
c) Determine phase voltage and phase current in case the load is in star connection (on the condition that performance of the load is unchanged).
d) Determine resistance, reactance and magnitude of the impedance of each phase for the cases that the load is delta-connected or star-connected respectively.

Find the active power absorbed by the three phase load (cos φ unchanged) when there is a fault in one phase interrupting current in that phase load completely

e) if the load is in delta connection,
f) if the load is in star connection with star point connected to the neutral conductor,
g) if the load is in star connection with star point open.

Time-independent magnetic fields

Problem 9.1

An electron is moving in a homogeneous magnetic field on a circular path (orbit). The circular path is perpendicular to the magnetic field lines. The magnetic induction is B = 1 mT. The velocity of the electron on the orbit is v = 3000 km/s. What are:

a) the force coming from the magnetic field F_m in N ?
b) the radius of the circular path r in cm ?
c) the angular velocity $\omega = v/r$ of the electron ?
d) the time T for a complete circulation of the electron ?

Problem 9.2

A d.c.-line (U = 200 kV high voltage, length l = 100 km, cross section A = 3,2 mm^2, distance between the two wires d = 1 m) made of copper (resistivity ρ_{Cu} = 16 $\Omega mm^2/km$) is short-circuited at the end. What is the force per meter (value and direction) on the two wires?

Problem 9.3

A current of I = 11,31 A flows through the following long and cylindrical arrangements. The slope of the magnetic field strength H(r) is to be calculated and to be displayed graphically in the range $0 \le r \le 6$ mm:

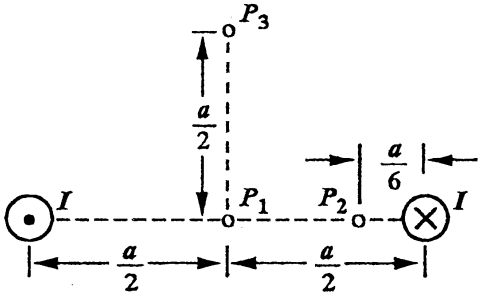
a) wire with 2 mm diameter
b) wire with 6 mm diameter
c) tube with 6 mm outer diameter and 1 mm wall thickness
d) coaxial arrangement consisting of the wire given in a) for the forward path and the tube given in c) for the return path.

Problem 9.4

A long cylindrical coil with N = 600 turns conducts a current of I = 2 A. Determine the magnetic field strength inside the coil, if the length of the coil is l = 60 cm.

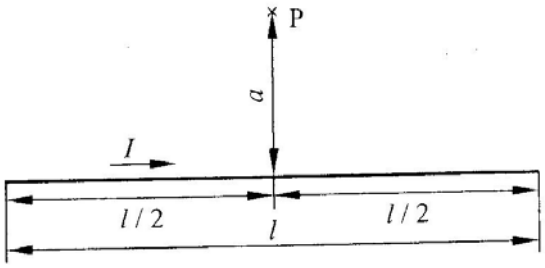
Problem 9.5

A current of I = 7,1 A flows through the given two-wire line (see sketch). The distance between the two wires is a = 56,5 cm. Calculate the values of the magnetic field strength H in the points P_1, P_2 and P_3 and draw the direction of the three field vectors in the sketch.



Problem 9.6

What is the length of a straight and linear conductor, so that the field strength in point P is 90% of the field strength of a conductor with infinite length?



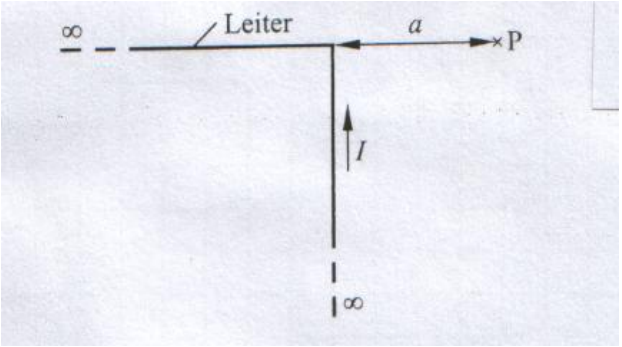
Problem 9.7

Calculate the magnetic field strength in the center of a quadratic, current carrying conductor loop.

Given: I = 2,5 A; side length s = 120 mm

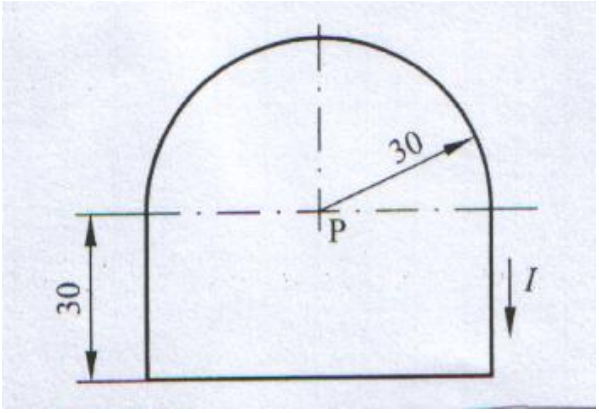
Problem 9.8

A conductor with infinite length has a right-angle corner. Determine the magnetic field strength in the point P.



Problem 9.9

The body of a coil has the design shown in the sketch. The coil consists of 200 turns. The current through the coil is I = 30 mA. What is the magnetic field strength in the point P?

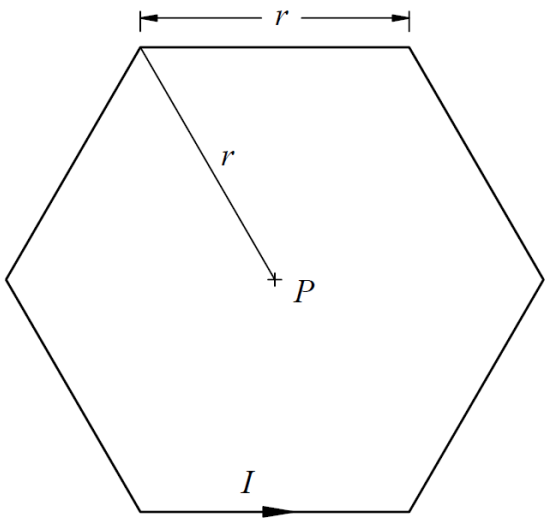


Problem 9.10

A conductor has the shape of a hexagon with six sides of equal length r = 4 cm.

A current I = 2 A is flowing through the conductor in the direction indicated in the sketch.

Calculate the magnetic field strength in the centre P of the hexagon.



Problem 9.11

A long, straight, and circular conductor carries a current I = 100 A. Around the conductor there is concentrically a circular ring with the quadratic cross-section A = 4 cm^2 and the mean radius r_m = 5 cm.

a) The ring consists of plastic (μ_r = 1). What is the magnetic field strength H in the center of the ring (at r = r_m)?
b) Calculate the magnetic flux density B in the ring center. From this determine approximately the entire magnetic flux Φ in the ring.
c) What is the magnetic flux in the ring as a result of an exact calculation?
d) What values do we get for the quantities in a) to c), if the ring is made of iron with the constant relative permeability μ_r = 100 ?

Value: $\mu_0 = 4\pi * 10^{-7}$ Vs/Am

Problem 9.12

A core made of sheet iron has an iron part's length of 57 cm and an air-gap's length of 1,42 mm. The magnetic characteristic can approximately be described with the following equation:

$$\frac{B}{T} = 1{,}6 - \frac{3{,}2}{2 + \frac{H}{A/cm}}$$

a) What is the necessary current linkage θ, if the induction shall be B = 1,4 T?
b) What is the relative permeability μ_r of the sheet iron in this operating point?
c) Draw the characteristic curve of the sheet iron's magnetization and the characteristic curve for the iron core with air-gap.

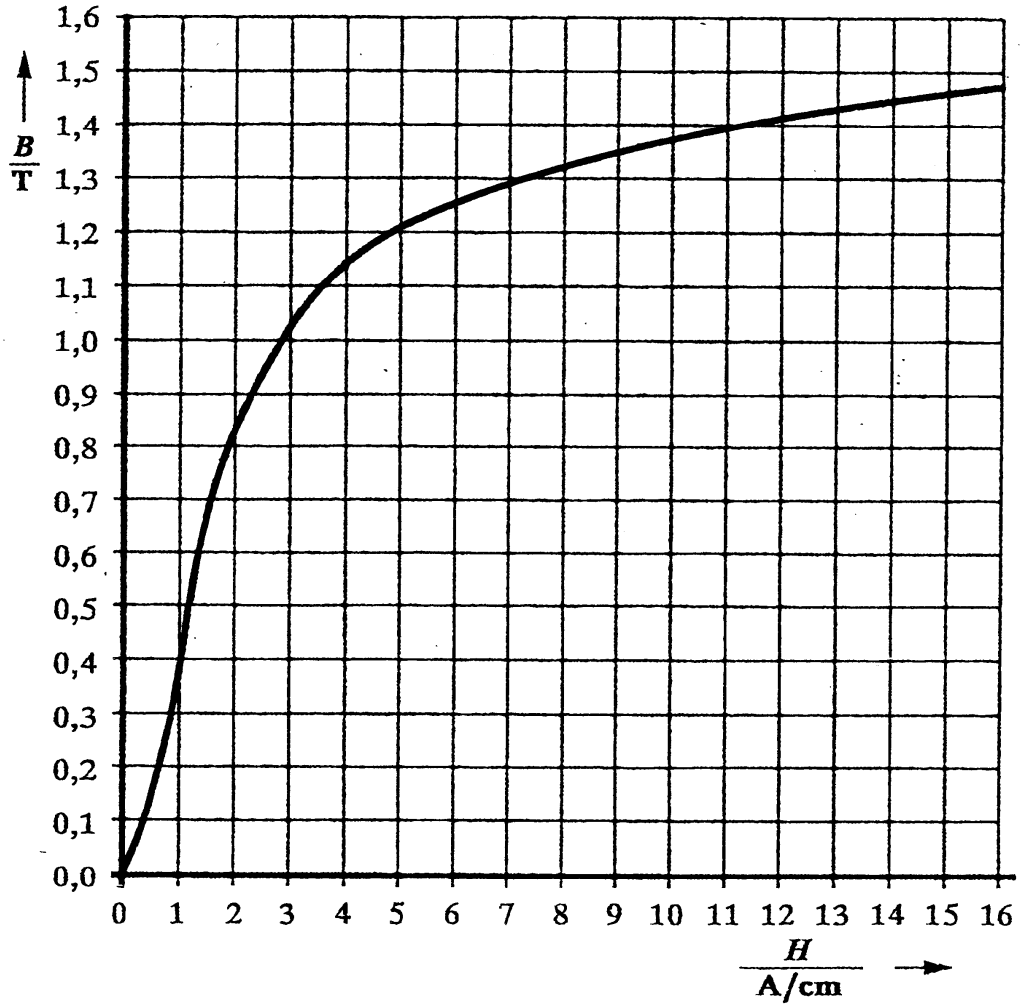
Problem 9.13

An iron ring with a cross-section of 5 cm^2 and a mean length of 120 cm is covered with 1000 turns of a copper wire. A current of 480 mA flows through the wire.

a) What is the mean magnetic field strength, the mean magnetic flux density and the magnetic flux in the iron ring?
b) What is the relative permeability μ_r in this operating point?
c) The iron ring is interrupted by an air-gap with a length of 0,4 mm. Dispersion at the air-gap can be disregarded.
   What are now the values of H, B and Φ in the iron ring?
d) What are the values of H, B and Φ in the air-gap?
e) What is now the effective relative permeability μ_r in the iron ring?

The magnetization characteristic curve of the used sheet iron is shown below.

(Magnetization characteristic curve for the problems 9.13 and 9.14)



Problem 9.14

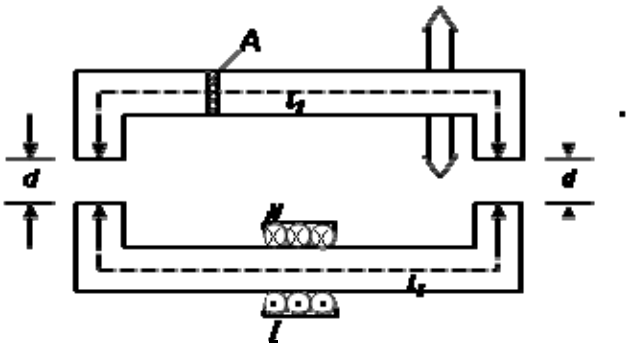
An iron sheet core (magnetization characteristic curve as given for problem 1.12) is covered with 2500 turns of a copper wire. The iron cross-section is 500 cm^2, the mean length of the field lines is 150 cm.

a) What is the current through the copper wire, if the magnetic flux density in iron shall be 1 T?
b) Now an air-gap of 2 mm length interrupts the iron. What is the current through the copper wire, to get a magnetic flux density of 1 T in the air-gap?
c) What would be the magnetic flux density in the air-gap, if still the current calculated ín a) would flow?
d) What are the magnetic resistances of the magnetic circuit for the cases a), b) and c) ?

Problem 9.15

An inductor has the design shown here. Given are:

- lenght of the iron core $l_1 = l_2 = 12{,}5$ cm
- iron's cross-section A = 4 cm^2
- iron's relative permeability $\mu_r = 5000$
- number of turns N = 50
- current I = 1 A.

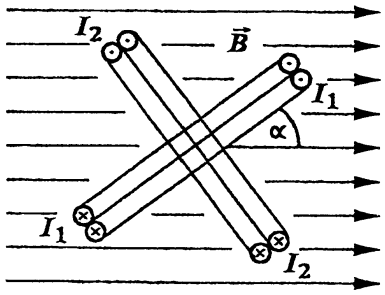


The length d of the two air-gaps can be varied between d = 0 and d = 1 mm by moving the upper leg of the iron core.

a) Determine a graphical description with numerical values for the dependency of the magnetic flux Φ on the air-gaps' length d, if the current is kept constant to I = 1 A.
b) For d = 1 mm calculate the magnetic field strength in iron and in the air-gaps.
c) If the inductor with the same cross-section A has no air-gaps, what would be the length of the iron core, if the magnetic resistance should be the same like for the inductor described above for d = 1 mm?

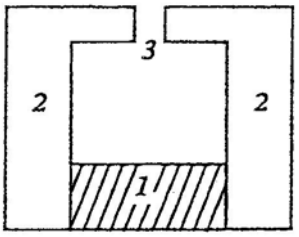
Problem 9.16

Two identical coils with rectangular size are mounted under a right angle, like shown in the sketch. The arrangement is rotatable around the axis without friction. It is brought to a homogeneous, timely constant magnetic field with the induction B. Both coils are electrically isolated against each other. One coil leads the current I_1, the other coil the current I_2. What is the angle α of the balance condition (equilibrium position)?

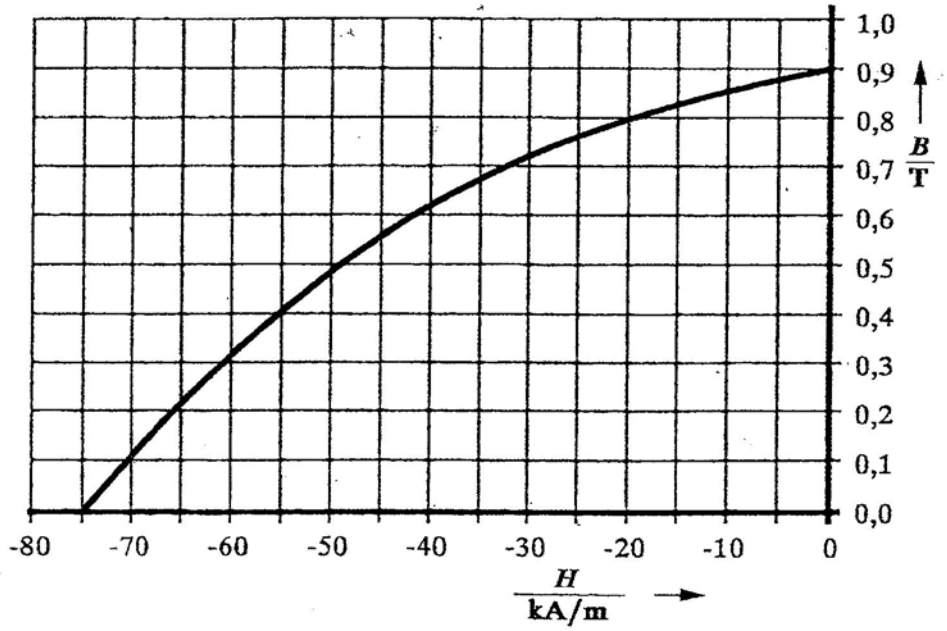


Problem 9.17

A permanent magnet (see sketch) consists of the magnet itself (1), two legs made of soft iron (2) and the operating air-gap (3). The magnetization characteristic curve is shown below. The operating air-gap has a cross-section of 5 cm^2 and a length of 2 mm. The dispersion at the air-gap can be disregarded. The magnetic resistance of the soft iron legs is negligible compared to the magnetic resistance of the air-gap.



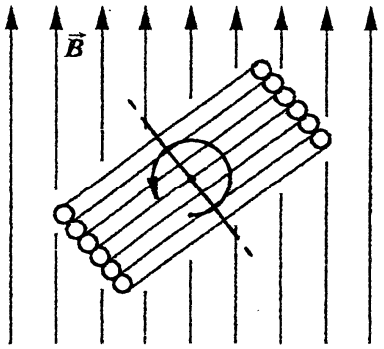
a) What is the magnetic field strength and the induction in the magnet and in the air-gap, if the permanent magnet has a length of 5 cm and a cross-section of 2 cm^2?
b) What is the cross-section and the length of the permanent magnet (1), if an induction of 0,66 T shall valid for the air-gap with a minimized material investment?
c) What is the maximum value for the induction in the air-gap, if the permanent magnet has a volume of 11,3 cm^3? What are the length and the cross-section for this condition?



Time-dependent magnetic fields

Problem 10.1

A coil with N = 22 turns is rotating with a speed of $n = 3000\ min^{-1}$ in a homogeneous magnetic field with the magnetic flux density B = 0,75 T. The coil has a cross-section of $A = 600\ cm^2$. The axis of rotation is perpendicular both to the coil's axis and to the magnetic field orientation (see sketch). At the time t = 0 the coil's axis is parallel to the field orientation.



a) Determine the time dependent voltage at the coil's open terminals. The directional arrows of voltage and magnetic flux are related according to the right-hand rule.
b) Determine the frequency and the angular frequency of the voltage.

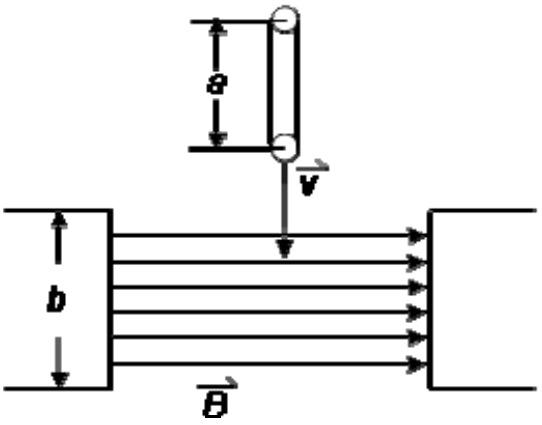
Problem 10.2

In order to receive a sinusoidal magnetic field, the receiver of a wireless personal paging system is connected to a frame antenna. The frame carries a coil with 100 turns and has an area of 10 cm^2 perpendicular to the magnetic field. For a reliable generation of the paging signal, the amplifier needs a voltage input of u_{max} = 20 mV. Determine the minimum value for the magnetic flux density's amplitude,

a) if the system's operating frequency is 5 kHz,
b) if the system's operating frequency is 50 kHz.

Problem 10.3

A rectangular coil with only one turn is moved through an air-gap with constant velocity v. In the air-gap there is a homogeneous magnetic field, which is assumed to be reduced to zero directly at the boundary. The air-gap has a quadratic cross-section. The length of the coil perpendicular to the displayed plane is greater than the length of the air-gap b. At time t = 0 the coil starts to penetrate in the magnetic field.

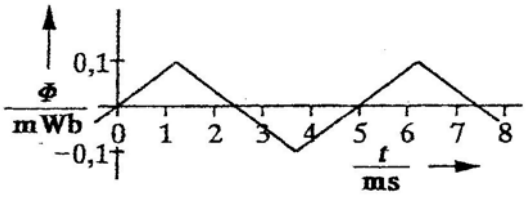


Given: B = 1 T; a = 1,5 cm; b = 2 cm; v = 0,5 m/s

Determine graphically the time dependent voltage induced in the coil for t > 0 (with numerical values).

Problem 10.4

The magnetic flux (time behaviour shown aside) passes through a coil with 125 turns. Calculate and draw the time dependent voltage appearing at the coil's terminals. The directional arrows of voltage and magnetic flux are related according to the right-hand rule.



Problem 10.5

1000 turns of a wire are wrapped evenly around a toroidal core (cross-section A = 2 cm^2, mean length l = 40 cm). What is the inductance of the winding, if a current of I = 40 mA flows and the core consists of the following materials:

a) plastic with a relative permeability of $\mu_r = 1$
b) iron with a relative permeability of $\mu_r = 200$
c) sheet iron (see magnetization characteristic curve for problem 1.12)?

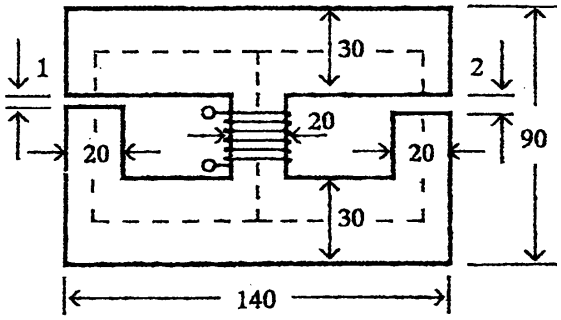
Problem 10.6

A ring-shaped coil has a quadratic cross-section. The inner radius is r_i = 2 cm, the outer radius is r_a = 3 cm. Determine firstly the inductance of the coil by a rough approximation, taking into account for the calculation only the mean field line. Then conduct secondly an exact calculation. What is the percentage of the error for the rough approximation?

Problem 10.7

Calculate the inductance of the given magnetic circuit, disregarding the dispersion (no stray fields).

The number of turns is N = 1635. The thickness of the iron core is 20 mm. All numbers in the sketch are in mm.

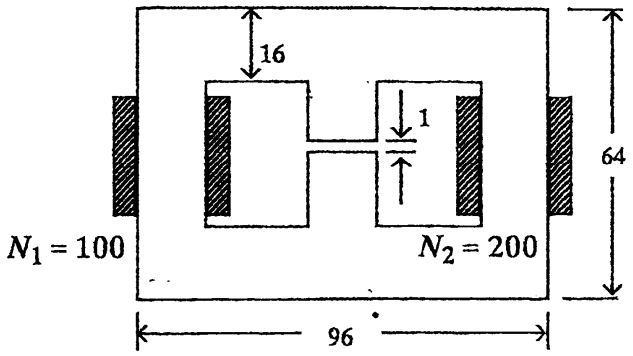


Permeability: $\mu_r = 200$
$\mu_0 = 4\pi * 10^{-7}$ Vs/Am

Problem 10.8

The transformer shown has a quadratic core cross-section. The relative permeability is $\mu_r = 497$. Stray fields can be ignored.

All numbers in the sketch are in mm.



a) Draw an analogical electrical equivalent circuit containing four resistances and calculate the values of the corresponding magnetic resistances.
b) Calculate the primary and the secondary inductance.
c) Calculate the coupling factor and the mutual inductance.
d) What are the values for the primary and secondary inductance and for the mutual inductance, if the core from problem 2.7 would be used?

Problem 10.9

A 35,5 cm long cylindrical coil with a cross-section of 11,3 cm^2 has 1000 turns. The core is made of plastic ($\mu_r = 1$). Determine the inductance and the energy stored in the coil, if a current of 500 mA flows.

Problem 10.10

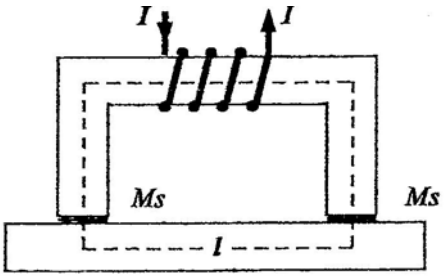
A sinusoidal voltage source with U = 100 V (rms-value) and the intrinsic resistance $R_i = 10\ \Omega$ is applied to an ideal transformer at the primary side. At the secondary side a load resistance $R_a = 1\ k\Omega$ is connected. With the use of the ideal transformer the maximum possible power should be transported to the load R_a. Determine the transmission ratio ü of the transformer and the maximum load power $P_{a/max}$.

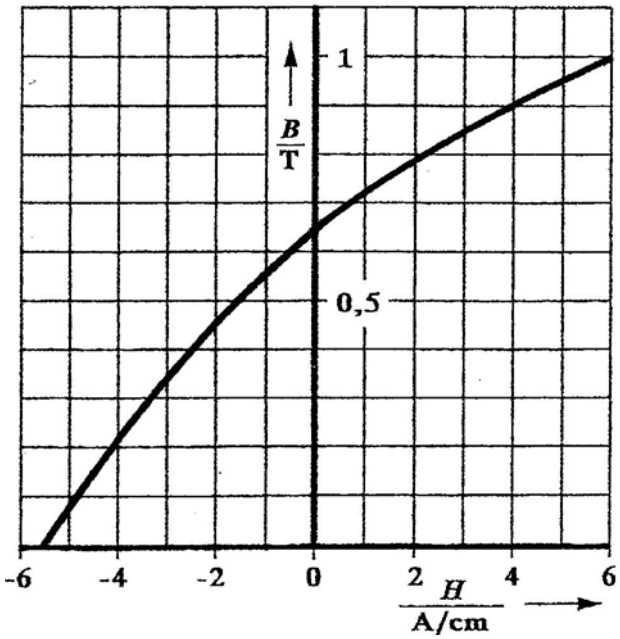
Problem 10.11

A transformer with the number of windings $N_1 = 1200$ and $N_2 = 600$, the magnetic resistance $R_m = 2*10^6$ A/Vs, and the coupling factor k = 0,9 is assumed to be lossless.

a) Calculate the elements of the equivalent circuit for the lossless transformer.
b) Using this equivalent circuit, determine:
   - the voltage ratio $\underline{U}_1/\underline{U}_2$ for the secondary open-circuit condition;
   - the current ratio $\underline{I}_1/\underline{I}_2$ for the secondary short-circuit condition.

Problem 10.12





The operating coil of the given hoisting magnet has N = 100 turns. The hoisting magnet has to lift an iron girder of the mass m = 104 kg. For safety reasons, the lifting force should be five times the weight of the iron girder. Between the magnet and the iron girder there is a 0,5 mm thick sheet of brass *Ms* on both sides (for a better performance of the hoisting magnet).
Magnet and iron girder have the same cross-section A = 100 cm^2. The magnetization characteristic shown aside is valid for magnet and iron girder as well. The entire length of the flux path through the iron is l = 2,33 m.

a) Determine the magnetic flux density and the magnetic field strength in the brass sheet.
b) Determine the magnetic field strength in the iron.
c) What is the current through the winding of the magnet?
d) What is the inductance of the winding?

12. Current displacement, skin effect

Problem 12.1

Given is an iron plate ($\kappa = 10^7$ S/m, $\mu_r = 5000$). A current I with a frequency of 50 Hz flows through the iron plate. The amplitude of the electric field strength at the surface of the plate is $E_y(0) = 20$ mV/m.

a) Determine the penetration depth δ.
b) Calculate the amplitude of the current density at the surface $J_y(0)$.
c) It is assumed, that for t = 0 the current density at the surface shows its maximum. For what values for the depth x_1 and x_2 do we get the first two zero points for the current density valid for the time t = T/4 ?
d) What current I flows in a strip with the width of $\Delta z = 10$ mm at the time t = 0 ?

Problem 12.2

Given are three rectangular conductors made of brass ($\kappa = 10^7$ S/m) with the width b = 50 mm. The three conductors have different thicknesses $d_1 = 0{,}05$ mm, $d_2 = 3$ mm, and $d_3 = 25$ mm. Calculate for f = 100 kHz:

a) the penetration depth δ;
b) the resistances per meter R' of the three conductors;
c) the current densities in the center of the three conductors $J_y(x = d/2)$, if the current density at the surface is $J_y(x = 0) = 1$ A/m^2.

Solutions

Basics

Problem 1.1: $\frac{W_{kin}}{kWh} = 10{,}7 \cdot 10^{-6} \cdot \frac{m}{t} \cdot \left(\frac{v}{km/h} \right)^2$

Problem 1.2: $\frac{G}{N} = 6{,}88 \cdot \left(\frac{d}{mm} \right)^2$

Problem 1.3: a) $\frac{v}{km/h} = 54 - 18 \cdot \frac{t}{s}$ b) $\frac{x}{m} = 15 \cdot \frac{t}{s} - 2{,}5 \cdot \left(\frac{t}{s} \right)^2$

Problem 1.4: a) $560 \cdot 10^3$ S/cm b) 0,0179 μΩ*m c) 8,4 km d) 2,4 cm^2

e) $\frac{R}{m\Omega} = 179 \cdot \frac{\frac{l}{km}}{\frac{A}{cm^2}}$ f) $\frac{G}{S} = 3{,}5 \cdot \frac{\frac{A}{cm^2}}{\frac{l}{km}}$

Analysis of DC circuits

Problem 2.1: a) $0{,}495 \frac{\Omega \cdot mm^2}{m}$ b) 9 V

Problem 2.2: a) 5.000 S b) 1,5 mm

Problem 2.3: 60 km

Problem 2.4: a) 546 Ω b) 109 V c) 83,6 °C

Problem 2.5: a) 529 Ω b) 30,5 Ω c) 7,53 A; 1,73 kW

Problem 2.6: 12 m

Problem 2.7 a) 7,72 A b) 16 A c) $\frac{R}{k\Omega} = 6561 \frac{1}{\left(\frac{U}{kV} \right)^3}$

Problem 2.8: c) 0,5 Ω d) 245 V e) 490 A

Problem 2.9: a) 10 Ω b) 1 A c) 10 V; 10 Ω d) 1 A; 100 mS

Problem 2.10: a) 4 A b) 96 V c) 384 W d) 16 W
e) 96 % f) 1 Ω g) 2,5 kW; 50 %

Problem 2.11: 93 %

Problem 2.12: b) 70,5 Ω; 105,8 Ω c) 300 W

Problem 2.13: Life time of 2W- lamp will be reduced

Problem 2.14: a) 6 A; 2 A; 4,5 A; 1,5 A
b) 128 W; 81 W; 27 W; 36 W; 48 W; 320 W; 640 W

Problem 2.15: a) 11,75 Ω b) 8 Ω c) 4,8 Ω

Problem 2.16: a) 20 mA; 19,2 V b) 384 mW c) 377,6 Ω; 622,4 Ω

Problem 2.17: a) 16,7 A b) 93,3 V

Problem 2.18: a) 24 mA b) 7 mm^2 c) 12,5 A d) 12,3V
e) 153,75 W f) 1,5 kW; 6 V; 250 A

Problem 2.19: 1,5 A; -0,5 A; 1 A; 200 V

Problem 2.20: a) 3 V; 30 A b) 2,625 V; 26,25 A

Problem 2.21: 0,5 A

Problem 2.22: 1 A; -1 A

Problem 2.23: a) 0,12 A; -1,5 A; -0,48 A b) 87,6 V; -14,4 V; 6 V

Problem 2.24: a) 6 V; 80 Ω b) 0,7 A; 50 mS

Problem 2.25: 300 mA

Problem 2.26: a) 12 V b) 4 V, 2 V

Problem 2.27: a) 100 mV c) 10 mΩ d) 0,5 W; 1 W
b) 0,1 kΩ; 0,3 kΩ; 0,5 kΩ; 1 kΩ; 3 kΩ; 5 kΩ; 10 kΩ; 30 kΩ; 50 kΩ; 100 kΩ; 300 kΩ

Problem 2.28: 120 Ω

Problem 2.29: a) 5 mV b) 10 mV c) 20 mV

Problem 2.30: a) 50 mA b) 95 Ω c) -1,1 %

Problem 2.31: a) 9,72 V b) 8,84 V c) 9,33 V; 9,20 V

Electrostatic Field

Problem 3.1:a) 0 b) -0,72 mN c) 1,44 mN
Problem 3.2:a) 0,1 mN b) swinging
Problem 3.3:± 6nC
Problem 3.4:512*10^9 V/m
Problem 3.5: $F = 0{,}914 \cdot \frac{Q^2}{4\pi \cdot \varepsilon_0 \cdot l^2}$ (repulsion)
Problem 3.6:a) E_1 = 67,5 kV/m; E_2 = 40,5 kV/m; E_3 = 54,0 kV/m b) 110,5 kV/m
c) 18°
Problem 3.7:see drawing; radial
Problem 3.8:a) 899,2 V/m b) 90 µN c) 0,525°
Problem 3.9: $\frac{D}{\frac{nAs}{m^2}} = 10 \cdot \frac{\ln\left[1+\left(\frac{r}{cm}\right)^3\right]}{\left(\frac{r}{cm}\right)^2}$; $\frac{E}{\frac{V}{m}} = 1130 \cdot \frac{\ln\left[1+\left(\frac{r}{cm}\right)^3\right]}{\left(\frac{r}{cm}\right)^2}$
Problem 3.10: $\frac{W}{J} = 10^{-2} \cdot \frac{a}{cm}$; $\frac{U}{kV} = 10 \cdot \frac{a}{cm}$
Problem 3.11: a) $E(r) = \frac{Q}{4\pi \cdot \varepsilon_0} \cdot \frac{1}{r^2}$; $\varphi(r) = \frac{Q}{4\pi \cdot \varepsilon_0} \cdot \left(\frac{1}{r} - \frac{1}{R_1}\right)$ for r < R_1, else E(r) = 0, φ(r) = 0
b) no change
c) r > R_2: $E(r) = \frac{Q}{4\pi \cdot \varepsilon_0} \cdot \frac{1}{r^2}$; $\varphi(r) = \frac{Q}{4\pi \cdot \varepsilon_0} \cdot \frac{1}{r}$
$R_1 \le r \le R_2$: $E(r) = 0$; $\varphi(r) = \frac{Q}{4\pi \cdot \varepsilon_0} \cdot \frac{1}{R_2}$
r < R_1: $E(r) = \frac{Q}{4\pi \cdot \varepsilon_0} \cdot \frac{1}{r^2}$; $\varphi(r) = \frac{Q}{4\pi \cdot \varepsilon_0} \cdot \left(\frac{1}{r} - \frac{1}{R_1} + \frac{1}{R_2}\right)$
Problem 3.12: a) $\frac{Q}{4/3 \cdot \pi \cdot R_1^3}$ b) r < R_1: $\frac{Q}{4\pi \cdot \varepsilon_0 \cdot R_1^3} \cdot r$; r > R_1: $\frac{Q}{4\pi \cdot \varepsilon_0 \cdot r^2}$
c) r > R_1: $\frac{Q}{4\pi \cdot \varepsilon_0} \cdot \frac{1}{r}$; r < R_1: $\frac{Q}{8\pi \cdot \varepsilon_0 \cdot R_1} \cdot \left(3 - \frac{r^2}{R_1^2}\right)$
Problem 3.13: a) 5 kV/cm b) 8,85 µAs/m² c) 885 pC d) 8,85 pF/cm²
Problem 3.14: a) 18 kV/cm b) 6,92 kV/cm
Problem 3.15: a) 7,96 nAs/m² b) 899 V/m c) 0,281 mN (attraction)
Problem 3.16: a) behind b) 150 pAs/m c) 5,97 nAs/m²; 674 V/m
Problem 3.17: a) 40 cm b) 1,33 c) 4 d) 3
Problem 3.18: 4,5 V, 3 V, 4 V, 8 V, 1,5 V, 3 V, 2 V, 4 V
Problem 3.19: a) 16 pF b) 0,8 nWs c) 1,6 nWs d) W_{mech} e) 20 V
Problem 3.20: a) 4,92 pF b) 40,2 pF c) C/pF = 4,92 + 3,53 h/cm
Problem 3.21: a) 8,85 Pa; 8,85 Ws/m³ b) 2,21 Pa; 2,21 Ws/m³
Problem 3.22: a) 0,25 … 4,75 m b) 29,3 pF c) 20,9 µC d) 7,43 Ws e) 39,8 Ws/m³
Problem 3.23: a) 6,64 pF; 250 V b) 250 V; 1500 V; 1750 V
c) 3320 nJ; 5810 nJ d) 5
Problem 3.24: a) $\frac{6\pi \cdot \varepsilon_0 \cdot h}{\ln 25}$ b) $\frac{3 \cdot U}{r_0 \cdot \varepsilon_0 \cdot \ln 25} \cdot \frac{1}{r/r_0}$ d) 5150 V
Problem 3.25: a) $\frac{E_1}{kV/m} = 99{,}9 \cdot \frac{1}{\left(r/cm\right)^2}$; $\frac{E_2}{kV/m} = 179{,}8 \cdot \frac{1}{\left(r/cm\right)^2}$ b) 138,8 V; 49,9 V
Problem 3.26: a) 50 V b) 26,3 V

Electrical Current Field

Problem 4.1: a) 200 A b) 100 A c) 0 A
Problem 4.2: 12 A
Problem 4.3: c) 16 A d) -8 A
Problem 4.4: b) $\frac{E}{kV/cm} = \frac{1}{r/cm}$ c) 644 V
Problem 4.5: 0,06 V/m
Problem 4.6: 2,21 nA/mm²; 22,1 kV/m
Problem 4.7: a) -38,4 V; 21,6 V b) -60 V; 21,6 V, 38,4 V
Problem 4.8: 0,0987 V/m; 3,4 A/mm²; 0,0987 V/m; 0,705 A/mm²
Problem 4.9: a) 2,5 Ω b) $U/kV = 93{,}9\left[\frac{1}{7{,}5} - \frac{1}{r/m}\right]$
c) 11 m; 20,8 m; 180,5 m d) 12,5 kV e) 1,47 kV
Problem 4.10: $\frac{\pi}{\kappa \cdot (r_a - r_i) \cdot \ln\left(r_a / r_i\right)}$
Problem 4.11: a) $\frac{\rho \cdot l}{\pi \cdot \delta \cdot (d - \delta)}$ b) $\frac{\rho}{2\pi \cdot l} \cdot \ln\left(\frac{d}{d - 2 \cdot \delta}\right)$ c) $r = \frac{d}{s} - \delta$; $\frac{U^2}{\rho \cdot \left(\frac{d}{2} - \delta\right)^2 \cdot \left[\ln\left(\frac{d}{d - 2 \cdot \delta}\right)\right]^2}$
Problem 4.12: a) $\frac{I}{2\pi \cdot r^2}$ b) $\frac{I}{2\pi \cdot \kappa_1} \cdot \frac{1}{r^2}$; $\frac{I}{2\pi \cdot \kappa_2} \cdot \frac{1}{r^2}$
c) $\frac{I}{2\pi} \cdot \left[\frac{1}{\kappa_1} \cdot \left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{\kappa_2}\left(\frac{1}{r_2} - \frac{1}{r_3}\right)\right]$ d) $\frac{1}{2\pi} \cdot \left[\frac{1}{\kappa_1} \cdot \left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{\kappa_2}\left(\frac{1}{r_2} - \frac{1}{r_3}\right)\right]$

Analysis of AC circuits

Problem 5.1: b) $u = 500V \cdot \sin(\omega t + \varphi)$; $\omega = 100 \cdot \pi\, s^{-1}$ c) 15° = 0,263 rad d) 4,16 ms
e) 12,11 ms
Problem 5.2: 1,195; 1,569
Problem 5.3: a) 1 kHz; 3 kHz b) 2,83 V; 2,83 V c) 4 V d) 3,395 V e) 1,178
Problem 5.4: a) 111 Hz b) 0,1 A d) 21,2 V
Problem 5.5: a) u_R = 35 V * sin(ωt); u_L = 55 V * cos(ωt) d) 65,2 V * sin(ωt + 57,5°)
Problem 5.6: a) 622 V b) 18,72 kΩ; 170 nF
Problem 5.7: 170 Ω * $e^{j67,5°}$; 1,3 A; -67,5°
Problem 5.8: 185 Ω; 80 mH
Problem 5.9: a) 21,4 Ω; -13,9 Ω; 7,5 Ω; 12,5 Ω * $e^{j36,9°}$
b) 80 mS; 64mS; -48 mS c) 88 V; 188,3 V; 122,3 V; 110 V
Problem 5.10: a) 1,44 mV b) -12,7 °
Problem 5.11: a) 100 Ω * $e^{j30°}$ b) 100 mA
Problem 5.12: 40 Ω; 4,51 µF
Problem 5.13: a) 22 Ω b) 110 Ω c) 107,8 Ω d) 343 mH
e) 78,5° f) 44 V; 215,6 V g) 550 Ω; 357 mH
Problem 5.14: a) 850 Hz; 372 Ω; 2,5; 10,7 b) 340 Hz; 79,4 Hz
c) 537 mA; 2,3 A; d) 200 V; 856 V
Problem 5.15: a) Series resonance circuit b) 5
Problem 5.16: a) 8 Ω * $e^{-j10°}$ b) 30 V * $e^{j10°}$; 11,7 V * $e^{-j26,5°}$; 5 A * $e^{j10°}$;1,85 A * $e^{-j26,5°}$
2,16 A * $e^{-j26,5°}$; 2,97 A * $e^{j63,5°}$ c) 200 VA; 197 W; -34,8 var
d) 2,93 mH e) 8,12 Ω
Problem 5.17: a) 2,56 kΩ; 78 pF b) 289 µH c) 1,56 A; 2,08 A d) 184,6 µH
e) 4,33 A; 6,66 kV; 17,31 kW
Problem 5.18: a) 226,7 Ω b) 70 W c) 5,898 µF d) 49 W; 50 var; 70 VA
Problem 5.19: a) 76,8 var; 92 VA b) 1,529 H; 66,25 Ω c) 100 V; 194 V
d) 3,314 µF e) 384 V f) 0,44 A; 1 g) 113,3°
Problem 5.20: a) 12,11 Mvar; 27,78 MVA; 11,25 Mvar; 18,75 MVA
b) 20 MW; 13,83 Mvar; 24,31 MVA; 0,823
Problem 5.21: 1249 µF

Loci

Problem 6.1: a) 500 kHz; 21,36 Ω; 11 b) 40 mA; 440 mA; 440 mA; 40 mA
c) vertical line through 4,26 mS; circuit through 0 and 235 Ω

Problem 6.2: a) 17 nF; 910 Ω b) 6 kHz c) 4,5 kHz; 8 kHz
d) (910 - j1386) Ω; (910 + j250) Ω
e) vertical line through 910 Ω f) circuit through 0 and 1,1 mS
g) shift to the right by 1/3 of the diameter
h) circuit through 682,5 Ω and 2730 Ω

Three-phase systems

Problem 7.1: a) 6,0 kW b) 5A; 7,5 A c) -36,87°; 0; 36,87°
e) 5,42 A; 9,80 A; 12,26 A

Problem 7.2: a) 20 A b) 6,929 kvar c) 10 Ω; 18,38 mH d) 13,33 Ω; 73,50 mH
e) 45,95 μF f) 137,84 μF g) 36 kW; 20,79 kvar

Problem 7.3: a) 6 kW; 8 kvar; 10 kVA b) 400 V; 25 A c) 230,9 V; 43,3 A
d) Δ: 9,6 Ω; 12,8 Ω; 16 Ω Stern: 3,2 Ω; 4,27 Ω; 5,33 Ω
e) 12 kW f) 12 kW g) 9 kW

Time-independent magnetic fields

Problem 9.1: a) $0{,}48*10^{-15}$ N b) 1,71 cm c) $1{,}75*10^{8}$ s^{-1} d) 35,8 ns

Problem 9.2: $8*10^{-3}$ N/m Repulsion

Problem 9.3: a) $\frac{H}{\frac{A}{cm}} = 18\frac{r}{mm}$ and $\frac{18}{\frac{r}{mm}}$ b) $\frac{H}{\frac{A}{cm}} = 2\frac{r}{mm}$ and $\frac{18}{\frac{r}{mm}}$

c) $\frac{H}{\frac{A}{cm}} = 0$ and $3{,}6\left[\frac{r}{mm} - \frac{4}{\frac{r}{mm}}\right]$ and $\frac{18}{\frac{r}{mm}}$

d) $\frac{H}{\frac{A}{cm}} = 18\frac{r}{mm}$ and $\frac{18}{\frac{r}{mm}}$ and $3{,}6\left[\frac{9}{\frac{r}{mm}} - \frac{r}{mm}\right]$ and 0

Problem 9.4: 20 A/cm

Problem 9.5: 8 A/m; 14,4 A/m; 4 A/m

Problem 9.6: I = 4,13 * a

Problem 9.7: 18,76 A/m

Problem 9.8: I/(4π*a)

Problem 9.9: 95 A/m

Problem 9.10: 27,57 A

Problem 9.11: a) 3,18 A/cm b) 0,4 mT; 0,16 μVs c) 0,162 μVs
d) 3,18 A/cm; 40 mT; 16 μVs; 16,2 μVs

Problem 9.12: a) 2380 A b) 796 c) $\frac{\frac{\Theta}{l}}{\frac{A}{cm}} = 19{,}82\frac{B}{T} - 2 + \frac{3{,}2}{1{,}6 - \frac{B}{T}}$

Problem 9.13: a) 4 A/cm; 1,15 T; 0,575 mVs b) 2290
c) 1,8 A/cm; 0,8 T; 0,4 mVs d) 6,37 kA/cm; 0,8T; 0,4 mVs
e) 3540

Problem 9.14: a) 168 mA b) 805 mA c) 0,2 T
d) 8,4 kA/Vs; 40,2 kA/Vs; 42 kA/Vs

Problem 9.15: b) 4,88 A/m; $24{,}4*10^{3}$ A/m c) 10,25 m

Problem 9.16: $\alpha = \arctan(I_1/I_2)$

Problem 9.17: a) -11kA/m; 0,85 T; 271 kA/m; 0,34 T b) 6 cm^2; 2,28 cm c) 0,6 T; 5,45 cm^2; 2,07 cm

Time-dependent magnetic fields

Problem 10.1: a) 311 V * sin(0,314 t/ms) b) 50 Hz c) 314 s^{-1}
Problem 10.2: a) 6,37 µT b) 0,637 µT
Problem 10.3: ± 10 mV
Problem 10.4: ± 10 V
Problem 10.5: a) 628 µH b) 126 mH c) 2 H
Problem 10.6: -1,35 %
Problem 10.7: 1 H
Problem 10.8: a) 294 kA/Vs; 3108 kA/Vs; 801 kA/Vs; b) 6,9 mH; 27,6 mH
c) 0,809; 11,2 mH d) 2,55 mH; 6,8 mH; 0,51 mH
Problem 10.9: 4 mH; 0,5 mWs
Problem 10.10: 0,1; 250 W
Problem 10.11: a) 0,720 H; 0,180 H; 0,324 H b) 2,22; 0,556
Problem 10.12: a) 0,8 T; 6,37 kA/m b) 2,2 A/cm c) 11,5 A d) 69,6 mH

Current displacement / skin-effect

Problem 12.1: a) 0,318 mm b) 0,2 A/mm^2 c) 0 mm; 1 mm d) 0,64 A
Problem 12.2: a) 0,503 mm b) 40 mΩ/m; 2 mΩ/m; 1,33 mΩ/m c) 1 A/m^2; 0,1 A/m^2; 0